Kinematic Analysis of an Exoskeleton Robot for Assisting Human Knee Motion

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**Abstract**
Due to the harmful changes in human lifestyle, different diseases like stroke and problems in musculoskeletal system are increasing noticeably. Wearable robots not only help these patients to rehabilitate their disabled organ and walk, but also have a significant mental effect on them. For the first time, the kinematic analysis of an exoskeletal orthosis designed for patients who have pain in one of their knees is presented using vector loop method. The proposed design for this exoskeleton has two linear actuators for resembling behavior of Quadriceps and Hamstring muscles, two of the most important muscles participating in human gait. The motion of the knee mechanism is simulated for a complete gait cycle. Obtaining velocities and accelerations of joints and links in this study will be useful in dynamic analysis of the mechanism.

1. Introduction

Nowadays, nearly 1% of the world population depends on wheelchairs for walking and movement. Despite the fact that wheelchairs have recently been improved, they have faced big problems like forcing people not to move and just sit for a long period of time. Studies indicate that knee is the most commonly injured area of the body and possibly accounts for 45% of all sports injuries [1]. One of the most common pathological conditions particularly among young people and athletes is anterior knee pain (AKP) [2]. Brain stroke is another major factor of causing unilateral pain in body. Person-oriented robots that run parallel to the wearer’s lower limbs like robotic leg exoskeletons and active orthoses are some cure options available to help manage pain and keep patients staying active. Wearable robots not only help these patients to rehabilitate their disabled organ and walk, but also have a significant mental effect on them.

Over the past few years many endeavors have been made to combine the human body and a robot into a single system. In 2007 an Active Leg Exoskeleton named ALEX was proposed and designed to assist right hemiparetic patients. In 2011 a passively supported gait training exoskeleton known as ALEX II was designed. Both ALEX devices have been designed to supply a controllable torque to a subject’s hip and knee joint [3-5]. Another commercial product is the knee orthosis Tibion PK100 (previously known as PowerKnee) that was unveiled by Tibion Bionic Technologies (USA) in 2009. Made of carbon fibers, the device is lightweight and portable [6]. WalkTrainer is another rehabilitation device that consists of a pelvic, a lower limb orthosis and a system of functional electrical stimulation that was proposed in 2009 [7-9].

Exoskeleton muscles have been modeled with various methods in literature. Pneumatic muscle actuators (PMA) that are light weight and have high power/weight and power/volume ratios as compared to the existing actuators [10], Pleated pneumatic artificial muscles [11], Magnetorheological Actuator [12] and electro-hydrostatic actuator that are capable of realizing backdrivability [13] are examples of models of artificial muscles.

Different mechanisms are introduced in the past to resemble knee joint. The simplest of all is proposed by Shamaei and Dollar that is a simple latched-spring mechanism. They showed that the torque-angle behavior of the knee can be approximated by a linear torsional spring. The focus of this work was on the performance of the knee in the weight acceptance stage of the stance phase of the gait. The gait speed and weight were chosen as two major parameters that affect the mechanical parameters of the knee [14].

Pyo et al. proposed kinematic design that is inspired by the knee biomechanics for an active knee orthosis. It is based on efficiently controlling the knee motions with hybrid actuations. The two actuators is implemented in proposed design; one as Hamstring and the other as Quadriceps [15].

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Lee and Guo formulated the kinematics of knee-joint orthosis designed for patients who have pain in one of their knees. The mechanism has two linear actuators for resembling behavior of Quadriceps and Hamstring muscles, two of the most important muscles participating in human gait. Using these two linear actuators has an improved achievement as compared to solely revolute joint actuation designs at the knee joint [15] due to having more complexity and simulating the knee motion with a more detailed mechanism. The novel kinematic analysis of an exoskeletal orthosis designed for patients who have pain in one of their knees is presented using vector loop method. The motion of the knee mechanism is simulated for a complete gait cycle. This proposed configuration is inspired by the knee biomechanics, therefore it provides more natural gait. It is expected that this new system enhances walking capacity for the patients [15].

2. Kinesiology Background

2.1. Anatomy

Knee is the largest joint in the human body. There are three bones involved in the knee joint [17]: The femur, the large bone in the upper leg; The tibia, the larger of the two bones in the lower leg and the patella, or kneecap (Figure 1).

Muscles are responsible for movements at joints. Two important muscles that apply the governing forces to the knee joint to provide necessary torques to support the human body weight and rotations of the knee joint in sagittal plane are Hamstring and Quadriceps. The Hamstring is attached between the pelvis and the tibia near back side of the knee joint. In addition, the length of Hamstring is the longest muscle within the part of lower limb. The position of Hamstring, however, is not very crucial in generating the torque at knee joint to support the body weight but provides fast angular velocity during swing phase [15]. Quadriceps is the only muscle which extends the knee [18], furthermore it is the most important muscle involved in stabilizing the knee joint [19]. The Quadriceps is found between the middle part of femur and the front side of tibia near knee joint. The behaviour of Quadriceps is different as compared with the Hamstring. The generated moment by the weight of human body is controlled by the compressive force of this muscle mainly. When the contraction of Quadriceps occur at knee joint, a reaction force by the patella ligament is produced, which generates a compression at the knee.

2.2. Gait Analysis

The gait cycle is defined as the time interval between two successive occurrences of one of the repetitive events of walking. It is generally convenient to use the instant at which one foot contacts the ground (initial contact) [18].

In Figure 2, it can be seen that the gait cycle is subdivided into seven periods, four of which occur in the stance phase, when the foot is on the ground, and three in the swing phase, when the foot is moving forward through the air. The stance phase, which is also called the support phase or contact phase, lasts from initial contact to toe off. It is subdivided into: 1. Loading response, 2. Mid-stance, 3. Terminal stance, 4. Pre-swing. The swing phase lasts from toe off to the next initial contact. It is subdivided into: 1. Initial swing, 2. Mid-swing, 3. Terminal swing [18].

The initial condition (IC) is defined by the moment of heel-strike. The phase of IC and TST (terminal stance) which take approximately 10% of the total gait cycle each, support the body by two legs. Therefore, the phase of heel-strikes as the starting point of stance and toe-off as the starting point of swing consume the largest muscle forces in a short time. Particularly, the transition states of gait cycle like IC that is the end point of swing phase and the PS that is next to TST and the end point of stance phase are difficult to the patient. These phases not only require power but also necessitate another capacity to attempt the heel-strike and the toe-off for the continuation of walk [20].

3. Mechanism Characteristics

The design of the knee orthosis under study in this paper is shown in Figure 3. This kinematic design of the knee orthosis that uses hybrid-actuating system with two actuators is proposed in [15].
The schematic design of the mechanism simply representing links and joints is shown in Figure 4. It can be divided into two parts. One part is the linear actuator as Hamstring (link 3) and the other part is the actuator as Quadriceps and the structure as patella (links 6 and 7).

The mechanism has two conditions depending on whether the flywheel is in contact with the patella part or not. Flywheels are often used to provide continuous energy in systems where the energy source is not continuous. In such cases, the flywheel stores energy when torque is applied by the energy source and it releases stored energy when the energy source is not applying torque to it. In the stance phase when the foot is on the ground, the torque produced by ground reaction force runs the flywheel and stores energy. In the swing phase when the foot is moving forward through the air, it releases the stored energy.

In the loading response and midstance portions of the gait cycle, the mechanism goes through contact state. To control the contact state, an actuator symbol is placed in Figure 6 to the contact point which in fact does not exist [15]. Using Grünler-Kutzbach criterion, we calculate the DOF of the non-contact and contact states as two and one, respectively.

4. Position Analysis

For the purpose of position analysis, we performed vector loop method on the mechanism. The analysis is done in the sagittal plane and investigated in both non-contact and contact states of the mechanism motion.

4.1. Non-Contact State

In non contact state the mechanism has six moving links (link 1 is fixed) and eight joints. The number of independent contours is $n_c = c - n = 8 - 6 = 2$, where $c$ is the number of joints and $n$ is the number of moving links [21]. We proposed two independent contours for the mechanism. The first Contour I contains the links 1 (a portion of it), 2, 3 and 4, while the second contour II contains the links 1 (the whole link), 4, 5, 6 and 7 as shown in Figure 5.

The equation associated with contour I is described as below

$$r_{KC} + r_{CB} + r_{BA} + r_{AK} = 0$$

$\Rightarrow r_{KC} \dot{e}_{KC} + r_{CB} \dot{e}_{CB} + r_{BA} \dot{e}_{BA} + r_{AK} \dot{e}_{AK} = 0$  \hspace{1cm} (1)
By combining Cartesian and polar coordinates, we have
\[
\dot{e}_r = 1 (\cos \theta t + \sin \theta j)
\] (2)

By substituting Eq. (2) in (1), we obtain
\[
\begin{align*}
r_{KC} (\cos \theta_{KC} + \sin \theta_{KC}) + r_{CB} (\cos \theta_{CB} + \sin \theta_{CB}) + \\
r_{BA} (\sin \theta_{BA} - \sin \theta_{BA}) - r_{AK} (\cos \theta_{AK} + \sin \theta_{AK}) &= 0
\end{align*}
\] (3)

In the above equation, the angles are defined between x axis and the corresponding vector (e.g. \(\theta_{CB}\) is the angle between x axis and link CB). We define two algebraic equations by projecting above equation onto the axes of a non-moving Cartesian frame:
\[
\begin{cases}
r_{KC} \cos \theta_{KC} + r_{CB} \cos \theta_{CB} + r_{BA} \cos \theta_{BA} + r_{AK} \cos \theta_{AK} = 0 \\
r_{KC} \sin \theta_{KC} + r_{CB} \sin \theta_{CB} + r_{BA} \sin \theta_{BA} + r_{AK} \sin \theta_{AK} = 0
\end{cases}
\] (4)

For contour II, we have the equation below
\[
r_{KD} + r_{DE} - r_{EF} - r_{HG} - r_{KH} = 0
\] (5)

In the same manner, we can define
\[
\begin{align*}
r_{KD} \cos \theta_{KD} + r_{DE} \cos \theta_{DE} + r_{EF} \cos \theta_{EF} - r_{HG} \cos \theta_{HG} - r_{KH} \cos \theta_{KH} &= 0 \\
r_{KD} \sin \theta_{KD} + r_{DE} \sin \theta_{DE} + r_{EF} \sin \theta_{EF} - r_{HG} \sin \theta_{HG} - r_{KH} \sin \theta_{KH} &= 0
\end{align*}
\] (6)

The design parameters in Eqs. (4) and (6) (\(l_{BA}, \theta_{CB}, \theta_{GF}\) and \(\theta_{HG}\)) are determined based on the practical design in Figure 3.

4.2. Contact-State

Contours I and II are solved using Eqs. (4) and (6) derived in previous section. Contour III is shown in Figure 6 and its supporting equation is as below
\[
\begin{align*}
r_{KH} + r_{HG} + r_{GS} - r_{KS} &= 0 \\
\Rightarrow r_{KH} \hat{e}_{KH} + r_{HG} \hat{e}_{HG} + r_{GS} \hat{e}_{GS} - r_{KS} \hat{e}_{KS} &= 0
\end{align*}
\] (7)

By substituting (2) in (7), we obtain
\[
\begin{align*}
r_{KH} (\cos \theta_{KH} + \sin \theta_{KH}) + r_{HG} (\cos \theta_{HG} + \sin \theta_{HG}) + \\
r_{GS} (\cos \theta_{GS} + \sin \theta_{GS}) - r_{KS} (\cos \theta_{KS} + \sin \theta_{KS}) &= 0
\end{align*}
\] (8)

We define Eq. (9) by projecting Eq. (8) on the vertical axis as
\[
r_{KH} \cos \theta_{KH} + r_{HG} \cos \theta_{HG} + r_{GS} \cos \theta_{GS} + r_{KS} \cos \theta_{KS} = 0
\] (9)

In the other hand, for maintaining the flywheel in contact with the patella we should keep link 8 perpendicular to link 6, thus
\[
\begin{align*}
(r_G - r_F) \times (r_G - r_S) &= 0 \\
(r_K - r_S) \times (r_G - r_S) &= 0
\end{align*}
\] (10) (11)

We assume \(r_i = (x_i, y_i, 0)\) and \(r_{GS} = |r_{GS}| = \sqrt{(x_S - x_G)^2 + (y_S - y_G)^2}\). By solving Eqs. (9), (10) and (11), \(x_i, y_i\), and \(\theta_{KS}\) can be determined.

5. Velocity and Acceleration Analysis

Velocity and acceleration analysis is initiated by link 4. The constant angular velocity of the driver link 4 is chosen 10 deg/sec based on the results obtained by Williamson and Andrews [22]. A Cartesian reference frame is placed on the mechanism as shown in Figure 7. The joint \(K\) is the origin of the reference frame and \(x_k = 0, y_k = 0\).

![Figure 7. Representation of Cartesian reference frame and angular velocity of link 4](image)

The angular velocity of link 4 is constant:
\[
\omega_4 = \omega_4 k = \frac{\pi n}{180} = \frac{\pi (10)}{180} \approx 0.1745 \text{ rad/sec}
\]

The angular acceleration of link 4 is
\[
\alpha_4 = \omega_4 = 0.
\]

The velocity of point \(A_4\) on link 4 is
\[
V_{A_4} = V_K + \omega_4 \times r_{KA}
\] (12)

Due to a rotational joint between links 4 and 3, the velocity of point \(A_3\) on link 3 is \(V_{A_3} = V_{A_4}\). The acceleration of \(A_4 = A_3\) is
\[
a_{A_4} = a_K + \alpha_4 \times r_{KA} - \alpha_4^2 r_{KA}
\] (13)

The Velocity of point \(A_2\) on link 2 is calculated in terms of velocity of point \(A_3\) on link 3 as
\[
V_{A_2} = V_{A_3} + V_{A_2|A_3} = V_{A_3} + V_{A_{23}}
\] (14)

where \(V_{A_{23}}\) is the relative velocity of \(A_2\) with respect to \(A_3\) on link 2. This relative velocity is parallel to the sliding direction \(AB\), \(V_{A_{23}} \parallel AB\), or...
\( V_{A23} = V_{A32} \left[ \cos(-\theta_{BA})i + \sin(-\theta_{BA})j \right] \)

(15)

The points A and C are on link 2 so

\( V_{A2} = V_{C} + \omega_{2} \times (r_{A} - r_{C}) \)

(16)

The angular velocity of link 2 is \( \omega_{2} = \omega_{2}k \). Eqs. (14), (15), and (16) give

\[
\begin{vmatrix}
  i & j & k \\
 0 & 0 & a_{o2} \\
 x_{A} - x_{C} & y_{A} - y_{C} & 0
\end{vmatrix}

= V_{A3} + V_{A23} \cos(-\theta_{BA})i + V_{A23} \sin(-\theta_{BA})j

(17)

Eq. (17) represents a vectorial equation with two scalar components on the x-axis and y-axis. \( \omega_{2} \) and \( V_{A23} \) can be solved by Eq. (18) as

\[
-\omega_{2}(y_{A} - y_{C}) = V_{A3} + V_{A23} \cos(-\theta_{BA})i
\]

\[
\omega_{2}(x_{A} - x_{C}) = V_{A3} + V_{A23} \sin(-\theta_{BA})j
\]

(18)

The angular velocity of link 3 is the same as the angular velocity of link 2, \( \omega_{2} = \omega_{3} \).

Acceleration of point A2 on link 2 is calculated in terms of the acceleration of point A3 on link 3.

\[
a_{A2} = a_{A3} + a_{rel}^{A2}A3 + a_{Cor}^{A2}A3 = a_{A3} + a_{A32} + a_{A33}
\]

(19)

where \( a_{rel}^{A2}A3 = a_{A23} \) is the relative acceleration of A2 with respect to A1 on link 2. This relative acceleration is parallel to the sliding direction \( AB, a_{A23} \parallel \ AB \), or

\[
a_{A23} = a_{A32} \left[ \cos(-\theta_{BA})i + \sin(-\theta_{BA})j \right]
\]

(20)

Coriolis acceleration of A2 relative to A3 is

\[
a_{Cor}^{A2}A3 = 2\omega_{2} \times V_{A23} = 2\omega_{2} \times V_{A23}
\]

(21)

The points A and C are on link 2, so

\[
a_{A2} = a_{c} + \alpha_{2} \times r_{CA} - \omega_{2}^{2}r_{CA}
\]

(22)

The angular acceleration of link 2 is \( \alpha_{2} = \alpha_{2}k \). Eqs. (19), (20), (21) and (22) give

\[
\begin{vmatrix}
  i & j & k \\
 0 & 0 & a_{o2} \\
 x_{A} - x_{C} & y_{A} - y_{C} & 0
\end{vmatrix}

= a_{A3} + a_{A32} \cos(-\theta_{BA})i + a_{A32} \sin(-\theta_{BA})j + 2a_{o2}V_{A23}

(23)

Eq. (23) represents a vectorial equation with two scalar components on the x-axis and y-axis. \( \alpha_{2} \) and \( a_{A23} \) can be solved by following

\[
\begin{align*}
-\alpha_{2}(y_{A} - y_{C}) - \omega_{2}^{2}(x_{A} - x_{C}) &= a_{A31} + a_{A32} \cos(\theta_{BA}) \\
-2\omega_{2}V_{A23} \sin(\theta_{BA}) &= a_{A32} \cos(\theta_{BA}) \\
\alpha_{2}(x_{A} - x_{C}) - \omega_{2}^{2}(y_{A} - y_{C}) &= a_{A31} + a_{A32} \sin(\theta_{BA}) + 2\omega_{2}V_{A23} \cos(\theta_{BA})
\end{align*}
\]

(24)

The angular acceleration of link 3 is the same as the angular acceleration of link 2, \( \alpha_{2} = \alpha_{3} \).

The velocity of point \( H_{4} \) on link 4 is

\[
V_{H} = V_{K} + \omega_{4} \times r_{KH}
\]

(25)

The velocity of point \( H_{5} \) on link 5 is \( V_{H_{5}} = V_{H_{4}} \) because between the links 4 and 5 there is a rotational joint.

The acceleration of \( H_{5} = H_{4} \) is

\[
a_{H} = a_{K} + \alpha_{4} \times r_{KH} - \omega_{2}^{2}r_{KH}
\]

(26)

The velocity of point G5 on link 5 is

\[
V_{G} = V_{H} + \omega_{5} \times r_{HG}
\]

(27)

The velocity of point G6 on link 6 is

\[
V_{G} = V_{F} + \omega_{6} \times r_{FG}
\]

(28)

The velocity of point G6 on link 6 is \( V_{G_{6}} = V_{G_{5}} \) because between the links 5 and 6 there is a rotational joint.

Figure 8. The relation between quadriceps actuator and knee angle [15]

According to the graph shown in Figure 8, the velocity of joint \( F \) relative to link 1 can be assumed to be 0.043 m/sec. Thus \( \omega_{5} \) and \( \omega_{6} \) can be obtained by Eqs. (27) and (28).

The acceleration of point G5 on link 5 is

\[
a_{G} = a_{H} + \alpha_{5} \times r_{FG} - \omega_{5}^{2}r_{FG}
\]

(29)

The acceleration of point G6 on link 6 is

\[
a_{G} = a_{F} + \alpha_{6} \times r_{FG} - \omega_{6}^{2}r_{FG}
\]

(30)

By Eqs. (29) and (30) the unknowns \( \alpha_{5} \) and \( \alpha_{6} \) can be obtained.

The accelerations of points B, O and N are defined by the following equations, respectively

\[
a_{B} = a_{C} + \alpha_{2} \times r_{CB} - \omega_{2}^{2}r_{CB}
\]

(31)

\[
a_{O} = a_{B} + \alpha_{5} \times r_{BO} - \omega_{5}^{2}r_{BO}
\]

(32)

\[
a_{N} = a_{H} \left( \frac{l_{EN}}{l_{KH}} \right)
\]

(33)
6. Motion Simulation and Results

Simulation has been carried out with MATLAB in the knee range of motion during a gait cycle. For obtaining the exact position of each joint at every moment the angle between femur and tibia is read from Figure 9. Figure 10 demonstrates some sample instants of the simulated knee motion. The joint trajectories in a complete gait cycle are represented in Figure 11.

![Figure 9. Sagittal plane joint angles (degrees) during a single gait cycle [18]](image)

**Figure 9.** Sagittal plane joint angles (degrees) during a single gait cycle [18]

![Figure 10. Simulated motion of knee mechanism in a single gait cycle](image)

**Figure 10.** Simulated motion of knee mechanism in a single gait cycle

![Figure 11. Joint trajectories in a complete gait cycle](image)

**Figure 11.** Joint trajectories in a complete gait cycle

In the following graphs, the behaviour of Hamstring muscle is demonstrated with more details. It can be observed in Figure 12 that linear velocity of this muscle increases up to almost 5 times as the knee angle reaches its maximum value. The linear acceleration of Hamstring is shown in Figure 13. As it is demonstrated the slope of this graph becomes steeper by increasing the knee angle. Figures 14 and 15 depict the angular velocity and acceleration of the link associated to this muscle, respectively. It is observable that angular velocity has an almost similar manner as the linear velocity during reaching the maximum angle of the knee.

![Figure 12. Relative velocity of joint A on link 2 with respect to joint A on link 3 (resembling Hamstring muscle)](image)

**Figure 12.** Relative velocity of joint A on link 2 with respect to joint A on link 3 (resembling Hamstring muscle)
7. Conclusions

In this paper, the novel kinematic analysis of an exoskeletal orthosis designed to help patients who have pain in one of their knees is presented using vector loop method. The motion of the knee mechanism is simulated for a complete gait cycle. The results are accurate and reliable according to the performed simulation and its analogy to knee motion in natural gait. The velocities and accelerations of the links associated to artificial muscles are calculated in order to become prepared for performing dynamic analysis on the mechanism in the further investigations. It is anticipated that this system will offer an enhanced walking capacity for the patients.

References