Computational Research Progress in Applied Science & Engineering
©PEARL publication, 2017

ISSN 2423-4591

CRPASE Vol. 03(04), 132-135, December 2017

Survey of Image De-noising using Wavelet Transform Combined with Thresholding Functions

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Keywords

Noise reduction,
Thresholding function,
Image de-noising,
Wavelet transform.

Abstract

Noise reduction is still a challenging problem for researchers. Many algorithms have been published in this subject and each finding has its own benefit and restriction. In this paper, a review of noise removing using some unique thresholding functions is conducted. All of these techniques used wavelet transform combined with thresholding functions for image de-noising. Their proposed thresholding functions have some properties like nonlinearity, continuity and smoothness. These functions were introduced to overcome the standard soft and hard thresholding functions and also to improve the performance analysis and enhance the visual quality of the images in terms of obtaining higher peak signal to noise ratio. Therefore, we can call these functions as improved version of standard soft and hard thresholding functions.

1. Introduction

Digital images play a significant role in daily events and also in areas of research and technology. Imperfect devices, troublesome during capturing, receiving and transmitting processes and meddling natural phenomena all degrade the data of interest [1]. Thus, analyzing the images may not be possible until we properly discard the noise from the images. It is necessary to apply efficient image de-noising techniques to remove the noise and enhance the visual quality of image.

Nowadays, wavelet based image de-noising has become very popular among the researchers investigating the image processing. Donoho and Johnstone proposed ideal spatial adaption by wavelet shrinkage in 1993 [2] and adaptive to unknown smoothness via wavelet shrinkage in 1995 [3]. Norouzzadeh and Rashidi suggested a new thresholding function in wavelet domain for image de-noising [4]. Chang et al. [5] proposed the adaptive wavelet thresholding for image de-noising and compression. Dong in 2013 introduced adaptive image de-noising using wavelet thresholding [6]. Anisimova et al. used the efficiency of wavelet coefficients thresholding techniques for multimedia and astronomical image de-noising [7]. Chen and Qian in 2011 [8] introduced de-noising of hyper-spectral imagery using principal component analysis and wavelet shrinkage. A transformation for ordering multispectral data in terms of image quality with implications for noise removal is proposed by Green et al. [9]. Thresholding neural network for adaptive noise reduction is introduced in a study conducted by Zhang [10]. Zhang and Desai used adaptive de-noising based on SURE risk [11]. Image de-noising in the wavelet domain using a new adaptive thresholding function has been introduced by Nasri and Nezamabadi-pour [12]. Guo et al. [13] used an efficient SVD-based method for image de-noising. Amiri Golilarz et al. [14] introduced the translation invariant wavelet based noise reduction using a new smooth nonlinear improved thresholding function. Wavelet image de-noising based on improved thresholding neural network and cycle spinning was proposed by Sahraeian et al. [15]. In addition, Amiri Golilarz and Demirel proposed thresholding neural network (TNN) based noise reduction with a new improved thresholding function [16].

Noise suppression using wavelet transform (WT) requires applying thresholding function with a suitable threshold value to keep the large coefficients (most important features) of the image and remove small noisy components. In this article, we presented a survey of some unique techniques for image de-noising. These methods were proposed to remove the noise from the images using WT combined with different thresholding functions.

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Received: 09 September 2017; Accepted: 20 November 2017

2. Noise Effect

According to Eq. (1) noise can affect the image and influence the visual quality.

$$y = x + n \qquad (1)$$

Here, x is the original image, n is the noise which can be additive white Gaussian noise (AWGN) with zero mean and standard deviation of σ and y is the noisy image.

The main objective in image de-noising is discarding the noise from images to improve the resolution, obtain higher peak signal to noise ratio (PSNR) and minimize the mean square error (MSE).

3. Standard Hard and Soft Thresholding Functions

3.1. Standard Hard Thresholding Function

In the hard thresholding, wavelet coefficients which their absolute values are greater than threshold (t) will be kept otherwise they will be set to zero. Eq. (2) shows the general formula for standard hard thresholding function [6].

$$Hard_h(x) = \begin{cases} x, & |x| > t \\ 0, & otherwise \end{cases} \qquad (2)$$

where $Hard_h(x)$ is the hard thresholding function, x is the wavelet coefficient and t is the universal threshold [2] which can be obtained using Eq. (4).

3.2. Standard Soft Thresholding Function

In soft thresholding, wavelet coefficients which their absolute values are greater than threshold (t) will be shrunk by (t) otherwise they become zero. Eq. (3) shows the general formula for standard soft thresholding function [6].

$$Soft_s(x) = \begin{cases} sgn(x).(|x| - t), & |\omega| \geq t \\ 0, & otherwise \end{cases} \qquad (3)$$

where $Soft_s(x)$ is soft thresholding function. Standard hard and soft thresholding functions are shown in Figure 1. It is clear that these thresholding functions are linear.

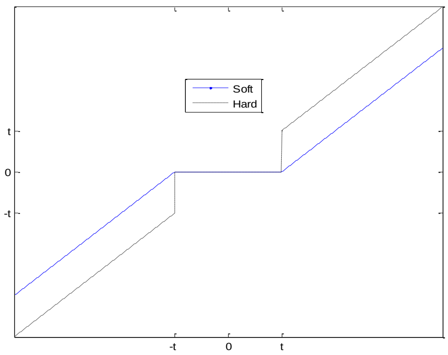

Figure 1. Hard and soft thresholding functions [16]

3.3. Wavelet Based Thresholding

In wavelet based thresholding technique, we have the following steps to obtain the output de-noised image

1. Applying discrete wavelet transform (DWT) to obtain the wavelet coefficients
2. Computing universal threshold value using the following relation

$$t = \sigma_n \sqrt{2log(a)} \qquad (4)$$

where $a = N^2$ is the size of image and σ_n stands for the standard deviation of noise in highest HH sub band in diagonal direction [6] defined by

$$\sigma_n = Median\,(|HH|)/0.6745 \qquad (5)$$

3. Applying standard hard or soft thresholding functions to get thresholded wavelet coefficients
4. Applying inverse discrete wavelet transform (IDWT) to acquire our desired output image.

Figure 2 shows the general block diagram of the de-noising process. As we see in this figure, applying DWT (2D-DWT) on input noisy image leads us to obtain wavelet coefficients (*HH*, *HL*, *LH* and *LL*) including detail and approximation coefficients. Detail coefficients carry the most important characteristics of image while noisy components are non-important coefficients which should be discarded from images to enhance the visual quality. To do so, we need a proper thresholding function to keep important coefficients and kill non-important coefficients. Applying thresholding function provides us with thresholded wavelet coefficients. Then, by applying IDWT on thresholded wavelet coefficients, we will get output de-noised image.

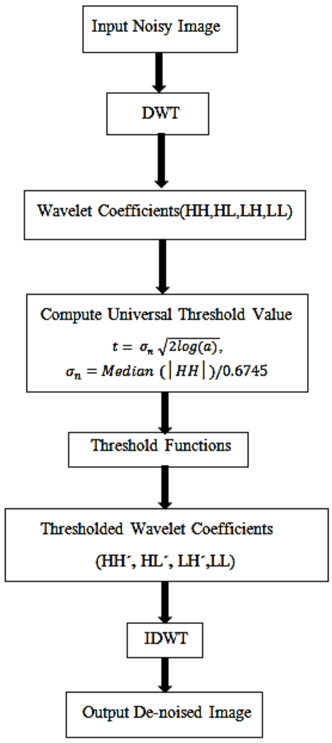

Figure 2. General block diagram of de-noising process

4. Nonlinear Thresholding Functions

4.1. Zhang's Improved Soft Thresholding Function

In 2001, Zhang presented a smooth nonlinear soft thresholding function [10] which is shown in Figure 3. This function can be constructed as follows

$$\tau_{st}(x,t)=x+\frac{1}{2}\left(\sqrt{(x-t)^2+\lambda}-\sqrt{(x+t)^2-\lambda}\right) \quad (6)$$

where $\lambda > 0$ is a user-defined (fixed) function parameter.

Figure 3 illustrates the improved soft thresholding functions for different values of λ. As we can see in the figure, when $\lambda = 0$, the threshold function becomes the standard soft thresholding function. While trying different values of λ, the threshold function becomes more smother.

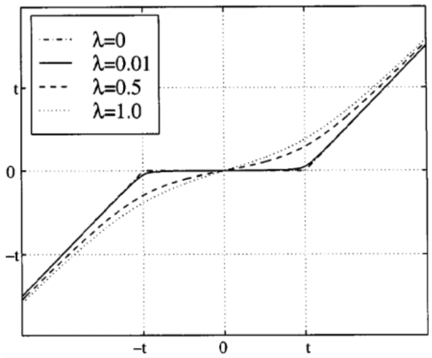

Figure 3. Zhang's improved soft thresholding function [10]

4.2. Zhang's Improved Hard Thresholding Function

In 2001, a nonlinear sigmoid based hard thresholding function was introduced by Zhang [10]. This function is given by Eq. (7) as

$$\tau_{ht}(x,t)=\left(\frac{1}{1+exp\{\frac{-x+t}{\mu}\}}-\frac{1}{1+exp\{\frac{-x-t}{\mu}\}}+1\right)x \quad (7)$$

in which $\mu > 0$ is a user-defined (fixed) function parameter.

When $\mu \to 0$, $\tau_{ht}(x,t)$ is just the standard hard-thresholding function $\tau_h(x,t)$, i.e., $\lim_{\mu \to 0} \tau_{ht}(x,t) = \tau_h(x,t)$ [10]. Figure 4 shows Zhang's improved hard thresholding function for different values of μ.

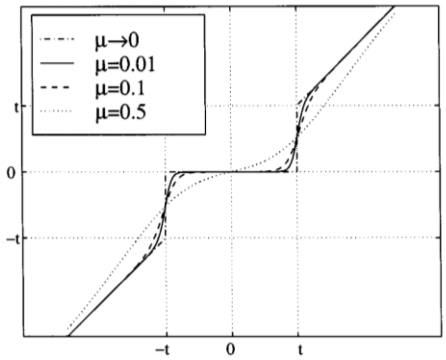

Figure 4. Zhang's improved Hard Thresholding Function [10]

4.3. Zhang's Nonlinear Soft-like Differentiable Thresholding Function

In 1998, Zhang and Desai proposed a nonlinear soft-like thresholding function motivated by differentiable sigmoid function which can be obtained by

$$\eta_k(x,t)=\begin{cases} x+t-\frac{t}{2k+1} & , \quad x<-t \\ \frac{1}{(2k+1)t^{2k}}x^{2k+1} & , \quad |x| \le t \\ x-t+\frac{t}{2k+1} & , \quad x>t \end{cases} \quad (8)$$

where $\eta_k(x,t)$ is the smooth nonlinear soft-like differentiable thresholding function and k is the positive integer. When k approaches to infinity, $\eta_k(x,t)$ becomes standard soft threshold function and when k approaches to zero, $\eta_k(x,t)$ becomes identity function $\eta_k(x,t) = x$. For other integer values of k, the shape of the threshold function becomes nonlinear and smooth. Figure 5 shows this thresholding function for different values of k.

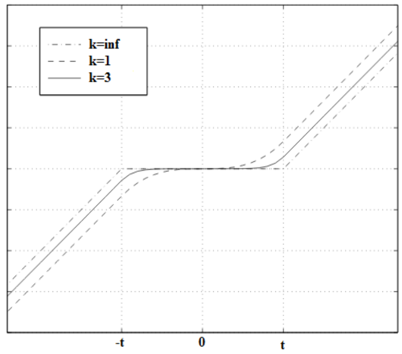

Figure 5. Zhang's nonlinear soft-like differentiable thresholding function for different values of k [11]

4.4. Nasri's Nonlinear Thresholding Function

Nasri and Nezamabadi-pour proposed a nonlinear thresholding function for image de-noising in 2009. This function can be constructed by Eq. (9) [12]. In this function, noisy coefficients (wavelet coefficients which their absolute values are less than threshold t) are tuned by a polynomial function [12]. Figure 6 shows this function with standard hard and soft thresholding functions in one graph.

$$\eta(x,t)=\begin{cases} x-0.5\frac{t^2}{x} , & |x| \ge t \\ 0.5\frac{x^3}{t^2} , & otherwise \end{cases} \quad (9)$$

with x and t defined as above.

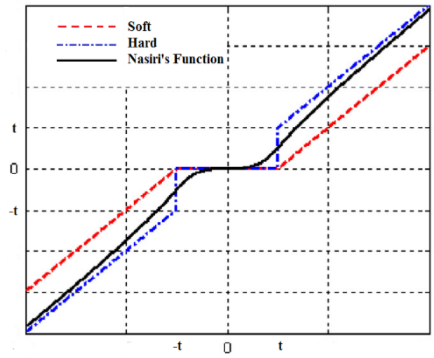

Figure 6. Hard, soft and Nasri's nonlinear thresholding function [12]

4.5. Noorbakhsh's Proposed Smooth Nonlinear Improved Thresholding Function

In 2017, Amiri Golilarz et al. [14] suggested a new type of nonlinear improved thresholding function for image de-noising which is given by Eq. (10). The main advantages of using this function are its smoothness, nonlinearity and data dependence properties. This function is shown in Figure 7.

$$f(x,t)=\begin{cases} sign(x)\left[|x|-\cos\left(\frac{\pi}{5}\left|\frac{t}{x}\right|\right)t\right], & |x|>t \\ 0.19\frac{x^3}{t^2} & , |x|<t \end{cases} \quad (10)$$

where x is wavelet coefficient and t is threshold value.

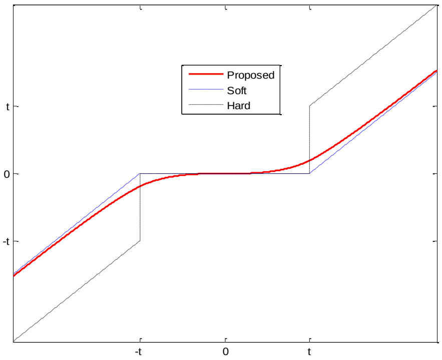

Figure 7. Hard, soft and Noorbakhsh's improved thresholding functions [14]

4.6. Sahraeian's Proposed Thresholding Function

In 2007, Sahraeian et al. [15] proposed a smooth differentiable hard thresholding function which can keep the good properties of standard hard thresholding function like preserving the detail characteristic of image edges [15]. Sahraeian's thresholding function can be constructed by

$$\eta_b(x,t)=\begin{cases} a(e^{b|x|}-1).sgn(x), & |x|\le t \\ (|x|+ce^{-b|x|}).sgn(x), & |x|>t \end{cases} \quad (11)$$

where parameter b determines the degree of the thresholding effect. Moreover, parameters a and c determine such that the continuity of the thresholding function and its derivative is preserved at the threshold value t [15]. Figure 8 shows this function for different values of b.

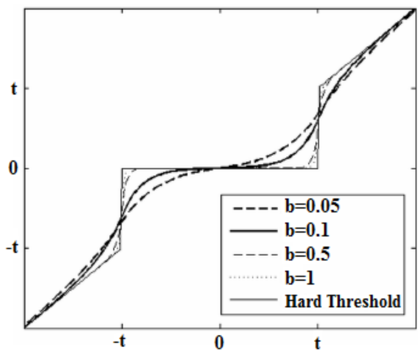

Figure 8. Sahraeian's proposed thresholding function for varying b values [15]

5. Conclusion

In this paper, a review of noise reduction using some unique thresholding functions is presented. In this regard, WT combined with thresholding functions were utilized to discard the noise from images. Linearity of standard soft threshold function and discontinuity of standard hard threshold function were some drawbacks of using these functions. Therefore, introducing functions to be smooth, continuous, nonlinear and differentiable improves the results in terms of acquiring the highest PSNR value and enhancing visual inspection.

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