



Research Article

The Effect of Particle Type and Distribution on Bending Analysis of Glass Particle Reinforced Composite Beams

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Keywords	Abstract
Particle reinforced composites, Functionally graded material, Finite element analysis, Bending deflection, Micro-CT based modelling.	In this work, the effect of particle type and distribution on the bending behavior of glass particle reinforced epoxy composite beams are studied analytically. Euler Bernoulli and Timoshenko Beam deflection results are compared with numerical deflections obtained by Fem Analysis. Bending analysis of particle reinforced composites are studied for graded distribution bottom to top. The composite elastic modulus variation with thickness is found according to micro-CT based modeling for different particle types and graded distribution. The effects of the graded distribution of particles are considered by functionally graded beam equation formulations. Local stress distribution along thickness is found by using finite element analysis. The relation between the stress results and particle type is discussed at a macro scale.

1. Introduction

Particle reinforced composites are popular in various applications with the resistance to impact to wearing. Functionally graded forms are extensively used in particle reinforced composites with the development of spatial distribution of fibers. The advantage of functionally graded material is to smooth transition of matrix and fiber dominated regions. The control of the property transition along the thickness is very important [1]. Functionally graded materials (FGM) are used at many kind of industry with the development of production methods. Powder metallurgy and centrifugal casting techniques are used for metal matrix FGM composites. Jang and coworkers are used glass and carbon particle reinforced thermoplastic FGM composites with controlling the change of distribution particles [2]. The distribution of properties along thickness are obtained by using fabric reinforcements at many studies in the literature [3,4].

Particle reinforced FGM thermoplastic composite production is increased with the development of newly developed production methods like selective laser sintering

[5]. The prediction of the material properties of these types of the composite is very important. Homogenization methods like Mori Tanaka are performed for material characterization of FGMs [6]. Homogenization based micromechanical approaches are increasingly used with the developments of computational methods like finite element based unit cell models or representative of volume elements [7]. Micro-CT equipped three-dimensional study of composite materials is alternative method with the implementation of FEA analysis. Micro-CT imaging technique is applied for both short fiber and continuous fiber composites in the literature [8,9]. Recently Homogenization based modeling of particle reinforced composites is obtained with the development of x-ray micro-CT to create a more effective characterization of spatially distributed particles [10].

Mathematical modeling of FGM beams have studied since early 2000s. Euler Bernoulli beam theory based elasticity solution of FGM beams are solved by Sankar [11]. Ding and coworkers are investigated the bending solutions of anisotropic FGM beams with arbitrary functions of thickness by elasticity solution [12]. The flexural bending of

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Received: 12 July 2020; Revised: 30 July 2020; Accepted: 10 August 2020

simply supported FGM beams are studied by using high order beam theories with varying gradation laws [13,14].

In this study micro-CT based finite element modeling is used to obtain the distribution of particles along thickness. The graded distribution of particles are modeled by using functionally graded beam formulations. The flexural behavior of beams are studied analytically and numerically.

2. Beam Theory

In this section bending formulations of the simply supported beam with concentrated load seen in figure 1 is given as follows

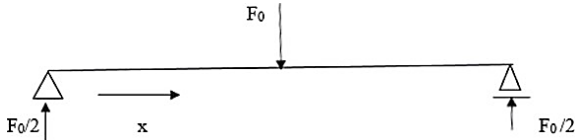


Figure 1. Simply supported beam under concentrated load at midspan

Force and Moment resultants can be obtained by substituting extensional, coupling, bending and transverse rigidities [15].

$$\begin{bmatrix} N_x \\ M_x \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} \\ B_{11} & D_{11} \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ k \end{bmatrix} \quad (1)$$

Here strain and curvature change is given

$$\varepsilon_0 = \frac{dU}{dx}, \quad k = -\frac{d^2w}{dx^2} \quad (2)$$

Beam moment may be defined by following way

$$M_x = B_{11} u_{,x} - D_{11} w_{,xx} \quad (3)$$

where here beam moment is also defined as

$$M_x = F_0 x/2 \quad (4)$$

External load variation along x direction is zero so it is defined as

$$\frac{dN}{dx} = 0, \quad A_{11}u_{,x} - B_{11}w_{,xxx} = 0 \quad (5)$$

$$u_{,x} = \frac{B_{11}w_{,xx}}{A_{11}} \quad (6)$$

If we add this term into Eq. (3) we obtain the following equation

$$w_{,xx} \left(\frac{B_{11}^2}{A_{11}} - D_{11} \right) = \frac{F_0 x}{2} \quad (7)$$

Here $C = \left(\frac{B_{11}^2}{A_{11}} - D_{11} \right)$ then

$$w_{,xx} = \frac{1}{C} \left(\frac{F_0 x^2}{4} + c_1 \right) \quad (8)$$

After integrating we obtain following equations

$$w_{,x} = \frac{1}{C} \left(\frac{F_0 x^2}{4} + c_1 \right) \quad (9)$$

$$w = \frac{1}{C} \left(\frac{F_0 x^3}{12} + C_1 x + C_2 \right) \quad (10)$$

$$u = \frac{B_{11}}{A_{11}} \left(\frac{1}{C} \frac{F_0 x^2}{4} + A_1 \right) \quad (11)$$

The integration constants are obtained by boundary conditions seen in Figure 1.

$$\text{For } x = 0 \quad u(0) = 0, w(0) = 0, \quad x = L \quad w(L) = 0$$

$$A_1 = 0, C_2 = 0$$

the deflection is symmetry at the point load and curvature is zero at that point

$$w_{,x} \left(\frac{a}{2} \right) = 0 \quad (12)$$

$$C_1 = \frac{F_0 a^2}{16} \quad (13)$$

$$w(x) = \frac{1}{C} \left(\frac{F_0 a^3}{48} \left(3 \left(\frac{x}{a} - 4 \left(\frac{x}{a} \right)^3 \right) \right) \right) \quad (14)$$

$$u = \frac{B_{11}}{A_{11}} \left(\frac{1}{C} \frac{F_0 x^2}{4} \right) \quad (15)$$

$$w(x) = \left(\frac{F_0 a^3}{48EI} \left(3 \left(\frac{x}{a} - 4 \left(\frac{x}{a} \right)^3 \right) \right) \right) + \frac{F_0 a}{2KG_{xz}bh} \left(\frac{x}{a} \right) \quad (16)$$

where rigidity $El = D_{11}$ for symmetric layup

$El = C$ for antisymmetric layup seen in eq (7).

Here K is shear correction factor generally $K = 5/6$, G_{xz} is the shear modulus of the beam.

A Numerical example is given for maximum deflection of isotropic epoxy beam in table 1 for the following values

$P=100N$, $L=80mm$, $b=5mm$, $h=5mm$, $E=1643 \text{ MPa}$.

3. Effective Material Properties of Composites

Micro-CT based finite element models (FEM) are used for material properties characterization of composites. Volume fraction variation along the thickness of composite beams are found from FEM. The rule of mixture homogenization procedure is used to find effective properties of beams [16].

Elastic modulus and shear modulus along z direction is given as follows

$$E(z) = E_1 V_1(z) + E_2 V_2(z) \quad (15)$$

$$G(z) = G_1 V_1(z) + G_2 V_2(z) \quad (16)$$

$$V_1(z) + V_2(z) = 1 \quad (17)$$

Here E_1, E_2, G_1, G_2 are constituents of the material,

$V_1(z), V_2(z)$ is denoted the volume fractions of constituents along the thickness direction.

According to Euler Bernoulli Theory, the extensional, coupling and bending rigidities for general form may be defined in the following form [17].

$$A(z) = \int_{-h/2}^{h/2} E(z) dz, \quad B(z) = \int_{-h/2}^{h/2} E(z) z dz,$$

$$D(z) = \int_{-h/2}^{h/2} E(z) z^2 dz \quad (18)$$

4. Finite Element Model in Numerical Analysis

The functionally graded beam is modeled by layer- based shell elements at finite element models. Shell elements are given good results for relatively thin-walled composite plates [18]. The finite element model and boundary conditions are

given in figure 2. Simply supported boundary conditions ($x=0, u=0, v=0, w=0, x=L, v=0, w=0$) are used in the analysis.

Table 1. Deflection values of the simply supported epoxy beam under midspan concentrated load

Euler Bernoulli Deflection (mm)	Timoshenko Beam Deflection (mm)	Anslys Solid Model (mm)	Anslys Shell Model (mm)
12.465	12.613	12.581	12.453

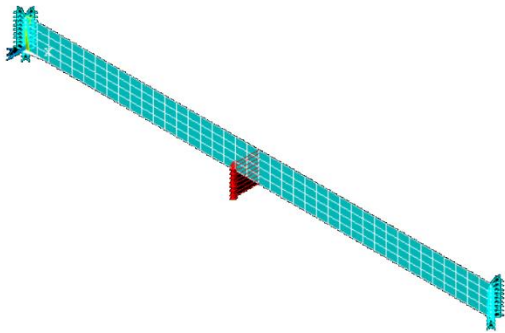


Figure 2. Finite element model and boundary conditions

Mesh convergence analysis is given at the following figure, 160 elements are used at the finite element model according to this study.

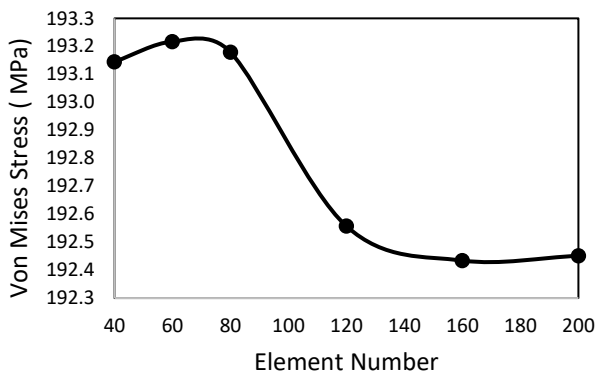


Figure 3. Mesh convergence analysis

5. Numerical Results

In our numerical study variation of volume fraction along thickness is taken from our previous study seen in figure 4. In that study, micro-CT image file is exported to bitmap image files then converted to voxel data. These data is implemented to FEM to obtain the distribution of particles along thickness. The glass particles were dispersed in the epoxy matrix with the %5 mass fraction and free to settle down at preparation of the composites. Elastic modulus results are obtained by rule of mixture formula given at eq.17 seen in figure 5.

It is seen that larger irregular particles are collapsed at the bottom side of composites. Particle size is effective for the smooth transition of particle distribution along thickness. Flake and rod type particles are more homogenous distribution than the other particles.

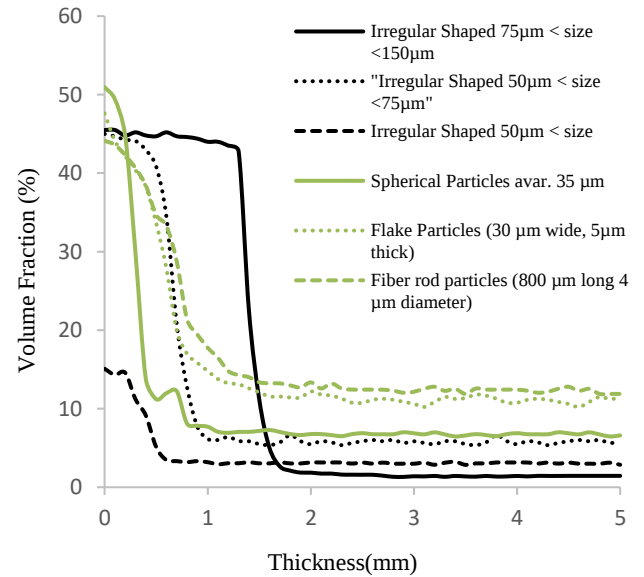


Figure 4. Variation of volume fraction along thickness

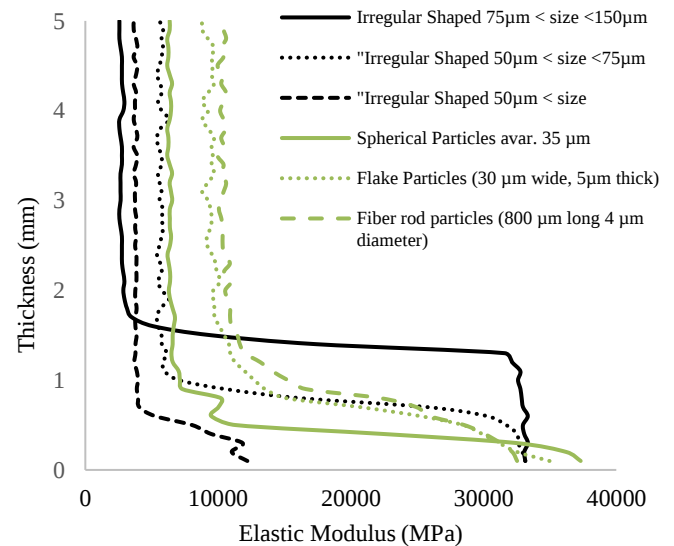
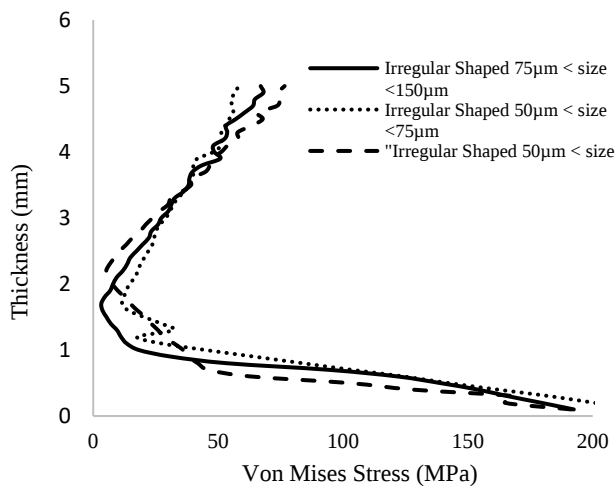


Figure 5. Elastic modulus variation with thickness

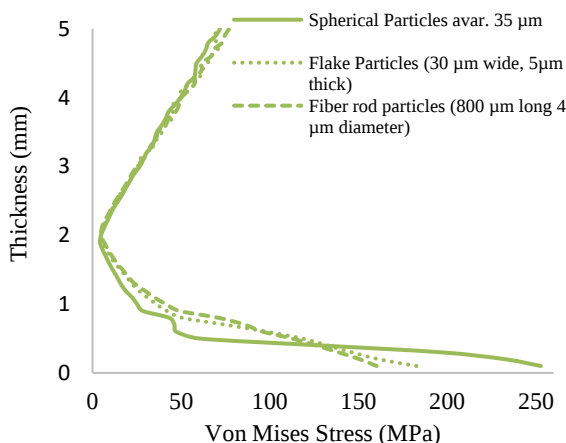
In the numerical and analytical analysis the elastic modulus of epoxy is $E_{ep} = 1.643$ GPa, the elastic modulus of glass particles are $E_{gl} = 70$ GPa and the Poisson ratio is 0.3. The deflection values are found for concentrated load at the midspan of the beam. The results are given for $P=100$ N load and ($L=80$ mm $b=5$ mm $h=5$ mm) beam dimensions.

Table 2. Deflection values of glass particle reinforced epoxy composites

Glass Particle Types	Euler Bernoulli Deflection Results w (mm)	Timoshenko Beam Deflection Results w (mm)	Timoshenko Beam Transverse Deflection Results (u) mm	(Ansys Shell Model) Deflection Results w (mm)	(Ansys Shell Model) Transverse Deflection Results u (mm)
Irregular Shaped (75 μm < size < 150 μm)	3.123	3.145	0.1599	3.150	0.159
Irregular Shaped (50 μm < size < 75 μm)	1.891	1.917	0.0616	1.921	0.0615
Irregular Shaped (50 μm < size)	3.9036	3.9578	0.0532	3.964	0.053
Spherical Particles	1.9224	1.9502	0.0451	1.953	0.0451
Flake Particles	1.4150	1.4343	0.0285	1.436	0.0285
Fiber Rod Particles	1.3104	1.3281	0.0246	1.330	0.0246

**Figure 6.** Von Mises stress variation along the thickness for irregular glass particle reinforced epoxy composites

Deflection results indicate that particle type and concentration affected overall composite stiffness. Local stresses are increasing at the bottom side of composite both for regular and irregular particles.

**Figure 7.** Von Mises stress variation along the thickness for glass particle reinforced epoxy composites

6. Conclusions

Deflection results indicate that larger glass particles are concentrated at the bottom and overall stiffness of composite is decreased. Particle type is seen as effective tool for distribution along thickness. Flake type and rod type particles are homogeneously distributed along with thickness at matrix according to the other particle types. Particle concentration at the bottom sides is the reason for the local stresses. Local stresses are predicted at macroscale with finite element method whereas finite element results with shell type elements are found in good agreement with Timoshenko Beam results. Local stresses must be eliminated to avoid composite failure for these types of products. In the future works particle type concentration, local stress and load transfer mechanism relations may be studied experimentally and numerically for optimization of these types of production methods.

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