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Research Article

On Maximum Likelihood Estimators of the Parameters of Three-Parameters Weibull Distribution Using Different Ranked Set Sampling Schemes

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Simple Random Sampling, Ranked Set Sampling, Extreme Ranked Set Sampling, Double Ranked Set Sampling, Estimation Parameter.

Abstract

Al-Saleh and Al-Kadiri first proposed double rank set sampling (DRSS). It seems that this ranked set sampling (RSS) modification can reduce the loss of RSS efficiency caused by ranking errors, and it is more effective than RSS and simple random sampling (SRS) to estimate the population mean. The proposed likelihood function is used to estimate the parameters of the three-Parameters Weibull distribution. Based on double ranked set sampling, extreme ranked set sampling, ranked set sampling (RSS) and simple random sampling (SRS) designs, the maximum likelihood estimator (MLE) is compared with the corresponding likelihood estimator. A simulation was carried out and the absolute relative biases, mean square error (MSE) and relative efficiency of different schemes were compared. It is found that, MSEs based SRS data has the largest MSEs comparing to RSS and its modifications schemes. This study revealed that DRSS technique has the superior over the rest of other sampling schemes. In almost all cases, DRSS has the smallest MSEs and largest efficiencies.

1. Introduction

The Weibull distribution is widely used in reliability and lifetime studies and proved an appropriate fit for most life data, except for data with non-monotonic empirical hazards. This type of data is often encountered in survival analysis, which makes it impossible for the Weibull model to analyze it. In many applications, α is assumed known (often $\alpha = 0$), for which results [1] guarantee the existence of a unique maximum likelihood estimator λ, β, and [2] the first of these is introduced tree parameter Weibull and concerned with asymptotic theory for maximum likelihood estimators. The cumulative distribution

function (CDF) of the three-parameter Weibull distribution is given by

$$F(x;\lambda,\beta,\alpha) = 1 - e^{-\lambda(\mathrm{x}-\alpha)^{\beta}} \quad (1)$$

where $\lambda > 0$, $\beta > 0$ and $\alpha < x$. The parameters λ, β and α are known as the scale, shape and location parameters, respectively. The corresponding probability density function (PDF) is

$$f(x;\lambda,\beta,\alpha) = \lambda\beta(\mathrm{x}-\alpha)^{\beta-1}e^{-\lambda(\mathrm{x}-\alpha)^{\beta}} \quad (2)$$

and the two parameters Weibull distribution is a special case of (2) when $\alpha = 1$.

2. Some Ranked Set Sampling Techniques

In this section, various sampling procedures for selection of units in the sample will be considered; brief descriptions of ranked set sampling (RSS), extreme ranked set sampling (ERSS) and double extreme ranked set sampling (DRSS) schemes will be introduced.

McIntyre, 1952 [3] proposed Ranked Set Sampling (RSS) to improve the estimation of the population mean, and

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the RSS method constitutes the sampling unit, because some measurements are not performed, the possibility of ranking errors increases. In order to overcome this problem, various modifications to RSS have been proposed, for example:[4] introduced extreme ranking set sampling (ERSS), [5] proposed median ranking set sampling (MRSS), The two-stage RSS design developed Double Ranked Set Sampling (DRSS) [6]. In addition to these studies, some authors have considered using RSS or its modification to estimate the parameters of a known distribution.

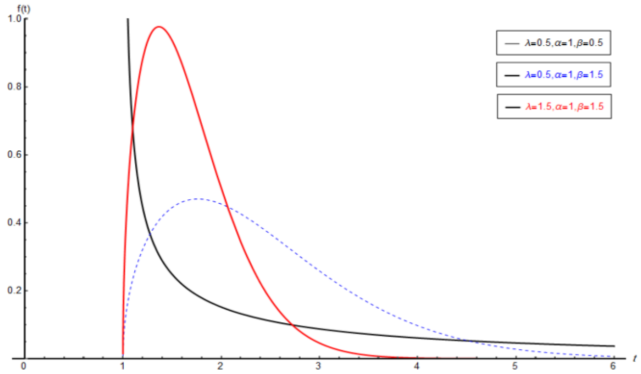

Figure 1. PDF of the EPGW distribution for different parameter values

For example, [7] studied Weibull and Pareto distributions, [8] introduced the maximum likelihood estimation of the modified Weibull distribution parameters using extreme ranking set sampling, [9], [10] and [11].and a k-stage RSS design uses n^{k+1} sample units from the target population to produce a sample of size n after k stages of ranking developed [12]. For more details, see [13], [14], [15] and [16], see more [17], [18] and [19].

2.1. Ranked Set Sampling

McIntyre proposed RSS technique as a useful procedure, the cost of quantifying all sampling units was high, but according to the characteristics of the survey, a small set of units can be easily ranked. Without actual quantification. The ranking criterion may be based on the value of accompanying variables or personal judgment, for example. A number of studies have proved that RSS is more efficient in estimating a large number of population parameters than SRS.

The RSS scheme can be described as follows:

Step 1: Randomly select m^2sample units from the population.

Step 2: Allocate the m2 selected units as randomly as possible into m sets, each of size m.

Step 3: By including the smallest ordered unit in the first group and the second smallest ordered unit in the second group, the sample is determined for actual quantification, This process will continue until the largest is selected from the last group The unit of sorting.

Step 4: Repeat steps 1 through 4 for r cycles to obtain a sample of size mr.

cycle 1			cycle 2			cycle 3		
$\boxed{X_{(11)1}}$	$X_{(12)1}$	$X_{(13)1}$	$\boxed{X_{(11)2}}$	$X_{(12)2}$	$X_{(13)2}$	$\boxed{X_{(11)3}}$	$X_{(12)3}$	$X_{(13)3}$
$X_{(21)1}$	$\boxed{X_{(22)1}}$	$X_{(23)1}$	$X_{(21)2}$	$\boxed{X_{(22)2}}$	$X_{(23)2}$	$X_{(21)3}$	$\boxed{X_{(22)3}}$	$X_{(23)3}$
$X_{(31)1}$	$X_{(32)1}$	$\boxed{X_{(33)1}}$	$X_{(31)2}$	$X_{(32)2}$	$\boxed{X_{(33)2}}$	$X_{(31)3}$	$X_{(32)3}$	$\boxed{X_{(33)3}}$

Figure 2. RSS design

Let $\{X_{(ii)s}, i = 1,2,..,m; s = 1,2,..,r\}$ be a ranked set sample where m is the set size and r is the number of cycles. Then the probability density function (PDF) of $X_{(ii)s}$ is given by

$$f_i(x_{(ii)j}) = C_1\,[F(x_{(ii)j})]^{i-1}.\,[1 - F(x_{(ii)j})]^{m-i} f(x_{(ii)j}) \qquad (3)$$

where $C_1 = \frac{m!}{(i-1)!(m-1)!}, , -\infty < x_{(ii)j} < \infty,$

using Eq. (3) the likelihood function corresponding to RSS scheme is given by:

$$L_{RSS}(\theta|x) = \prod_{j=1}^{r}\prod_{i=1}^{m} C_1\, f(x_{(ii)j};\theta)[F(x_{(ii)j};\theta)]^{i-1} \cdot [1 - F(x_{(ii)j};\theta)]^{m-i} \qquad (4)$$

2.2. Extreme Ranked Set Sampling

[4] introduced a modification of McIntyre's RSS scheme to produce other sampling schemes the extreme ranked set sampling (ERSS) which does not need a complete ranking as for RSS. They investigated ERSS by quantifying the smallest and largest order statistics instead of detailed ranking. The ERSS procedure can be summarized as follows

Step (1): Select m random samples of size m units from the population.

Step (2): Rank the units within each sample with respect to a variable of interest by visual inspection or any other cost free method.

Step (3): If the sample size n is odd, select from $\frac{m-1}{2}$ samples the smallest unit, from the other $\frac{m-1}{2}$ the largest unit and for the last sample select the median of the sample for actual measurement.

$$\begin{bmatrix} x_{(1\,1)j} & x_{(1\,2)j} & \cdots & x_{(1\,\frac{m+1}{2})j} & \cdots & x_{(1\,m)j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{(\frac{m-1}{2}\,1)j} & x_{(\frac{m-1}{2}\,2)j} & \cdots & x_{(\frac{m-1}{2}\,\frac{m+1}{2})j} & \cdots & x_{(\frac{m-1}{2}\,m)j} \\ x_{(\frac{m+1}{2}\,1)j} & x_{(\frac{m+1}{2}\,2)j} & \cdots & x_{(\frac{m+1}{2}\,\frac{m+1}{2})j} & \cdots & x_{(\frac{m+1}{2}\,m)j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{(m-1\,1)j} & x_{(m-1\,2)j} & \cdots & x_{(m-1\,\frac{m+1}{2})j} & \cdots & x_{(m-1\,m)j} \\ x_{(m\,1)j} & x_{(m\,2)j} & \cdots & x_{(m\,\frac{m+1}{2})j} & \cdots & x_{(m\,m)j} \end{bmatrix}$$

$$\xrightarrow{for\ m\ odd\ and\ r\ times} \begin{bmatrix} \boxed{x_{(1\,1)j}} & x_{(1\,2)j} & \cdots & x_{(1\,\frac{m+1}{2})j} & \cdots & x_{(1\,m)j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \boxed{x_{(\frac{m-1}{2}\,1)j}} & x_{(\frac{m-1}{2}\,2)j} & \cdots & x_{(\frac{m-1}{2}\,\frac{m+1}{2})j} & \cdots & x_{(\frac{m-1}{2}\,m)j} \\ x_{(\frac{m+1}{2}\,1)j} & x_{(\frac{m+1}{2}\,2)j} & \cdots & x_{(\frac{m+1}{2}\,\frac{m+1}{2})j} & \cdots & \boxed{x_{(\frac{m+1}{2}\,m)j}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{(m-1\,1)j} & x_{(m-1\,2)j} & \cdots & x_{(m-1\,\frac{m+1}{2})j} & \cdots & \boxed{x_{(m-1\,m)j}} \\ x_{(m\,1)j} & x_{(m\,2)j} & \cdots & \boxed{x_{(m\,\frac{m+1}{2})j}} & \cdots & x_{(m\,m)j} \end{bmatrix}$$

Figure 3. ERSS Design in Case of Odd Sample Size

The PDFs of the ERSS in case of odd sample size will be as follows:

$$f_m(x_{(i1)j}) = m\, f(x_{(i1)j};\theta)\,[1 - F(x_{(i1)j};\theta)\,]^{m-1},$$
$$-\infty < x_{(i1)j} < \infty \qquad (5)$$

where $1 \le i \le q,\ q = \frac{m-1}{2}$

$$f_m(x_{(im)j}) = m[f(x_{(im)j};\theta)]^{n-1}[1 - F(x_{(im)j};\theta)\,]^{m-1},$$
$$-\infty < x_{(im)j} < \infty. \qquad (6)$$

where $q + 1 \le i \le m - 1,$ and

$$f_m\left(x_{\left(m\ \frac{m+1}{2}\right)j}\right) = \frac{m!}{\left[\left(\frac{m+1}{2} - 1\right)!\right]^2} f\left(x_{\left(m\ \frac{m+1}{2}\right)j};\theta\right) \cdot \left(F\left(x_{\left(m\ \frac{m+1}{2}\right)j};\theta\right)\left(1 - F\left(x_{\left(m\ \frac{m+1}{2}\right)j};\theta\right)\right)\right)^{\frac{m-1}{2}}$$
$$-\infty < x_{\left(m\ \frac{m+1}{2}\right)j} < \infty. \quad (7)$$

where $x_{(i1)j}$is the smallest order statistic from the i^{th} set of the j^{th} cycle, $x_{(im)j}$ is the largest order statistic from the i^{th} set of the j^{th} cycle and $x_{\left(m\ \frac{m+1}{2}\right)j}$ is the median order statistic from the m^{th} set of the j^{th} cycle.

The Likelihood function corresponding to ERSS scheme for m is odd and with r cycles is given as follows:

$$L_{ERSS}(\theta) \propto \left[\prod_{j=1}^{r}\prod_{i=1}^{q} m\, f(x_{(i1)j};\theta)\ [F(x_{(i1)j};\theta)\,]^{m-1}\right] \cdot \left[\prod_{j=1}^{r}\prod_{i=q+1}^{m-1} m\, f(x_{(im)i,j};\theta)\ [1 - F(x_{(im)j};\theta)\,]^{m-1}\right] \cdot \left[f\left(x_{\left(m\frac{m+1}{2}\right)j};\theta\right)\left(F\left(x_{\left(m\frac{m+1}{2}\right)j};\theta\right)\left(1 - F\left(x_{\left(\frac{m+1}{2}m\right)j};\theta\right)\right)\right)^{\frac{m-1}{2}}\right]$$

Step (4): If the sample size is even, select from $\frac{m}{2}$ samples the smallest unit and from the other $\frac{m}{2}$ samples the largest unit for actual measurement**.**

$$\begin{bmatrix} x^{(1)}_{(1\,1)} & x^{(1)}_{(1\,2)} & \cdots & x^{(1)}_{(1\,m)} \\ \vdots & & \vdots & \vdots \\ x^{(1)}_{(\frac{m}{2}\,1)} & x^{(1)}_{(\frac{m}{2}\,2)} & \cdots & x^{(1)}_{(\frac{m}{2}\,m)} \\ x^{(1)}_{(\frac{m}{2}+1\,1)} & x^{(1)}_{(\frac{m}{2}+1\,2)} & \cdots & x^{(1)}_{(\frac{m}{2}+1\,m)} \\ \vdots & \vdots & \vdots & \vdots \\ x^{(1)}_{(m\,1)} & x^{(1)}_{(m\,2)} & \cdots & x^{(1)}_{(m\,m)} \end{bmatrix} \xrightarrow{for\ m\ even\ and\ r\ times} \begin{bmatrix} \boxed{x^{(1)}_{(1\,1)}} & x^{(1)}_{(1\,2)} & \cdots & x^{(1)}_{(1\,m)} \\ \vdots & \vdots & \vdots & \vdots \\ \boxed{x^{(1)}_{(\frac{m}{2}\,1)}} & x^{(1)}_{(\frac{m}{2}\,2)} & \cdots & x^{(1)}_{(\frac{m}{2}\,m)} \\ x^{(1)}_{(\frac{m}{2}+1\,1)} & x^{(1)}_{(\frac{m}{2}+1\,2)} & \cdots & \boxed{x^{(1)}_{(\frac{m}{2}+1\,m)}} \\ \vdots & \vdots & \vdots & \vdots \\ x^{(1)}_{(m\,1)} & x^{(1)}_{(m\,2)} & \cdots & \boxed{x^{(1)}_{(m\,m)}} \end{bmatrix}$$

Figure 4. ERSS Design in Case of Even Sample Size.

The PDFs of the ERSS in case of even sample size are defined as in Eq. (5) and Eq. (6) and the Likelihood function corresponding to ERSS scheme for even set sizes ($m = 2p$) and with r cycles introduced by [8] is given as follows:

$$L_{ERSS}(\theta) \propto \left[\prod_{j=1}^{r}\prod_{i=1}^{p} m\, f(x_{(i1)j};\theta)\ [1 - F(x_{(1i)j};\theta)\,]^{m-1}\right] \cdot \left[\prod_{j=1}^{r}\prod_{i=p+1}^{m} m\, f(x_{(im)j};\theta)\ [F(x_{(im)i,j};\theta)\,]^{m-1}\right]$$

Step (5): The cycle may be repeated r times to get $n = mr$ units from ERSS data.

2.3. Double Ranked Set Sampling

The best known multistage RSS procedure is DRSS design. DRSS is a two stage design was proposed by [6], which can be detailed as follows:

Step 1: Select m^3 elements from the target population and divide these elements randomly into m sets (of size m^2).

Step 2: Select a sample of size m in each set using RSS method.

Step 3: Apply the RSS procedure again to elements selected in step 2 to obtain a DRSS of size m.

Step 4: The cycle may be repeated r times.

$$\begin{bmatrix} x_{(11)1} & \cdots & x_{(1\,m)1} \\ \vdots & \ddots & \vdots \\ x_{(m1)1} & \cdots & x_{(mm)1} \end{bmatrix}, \begin{bmatrix} x_{(11)2} & \cdots & x_{(1\,m)2} \\ \vdots & \ddots & \vdots \\ x_{(m1)2} & \cdots & x_{(mm)2} \end{bmatrix}, \begin{bmatrix} x_{(11)3} & \cdots & x_{(1\,m)3} \\ \vdots & \ddots & \vdots \\ x_{(m1)3} & \cdots & x_{(mm)3} \end{bmatrix} \quad (8)$$
$$and \ldots \begin{bmatrix} x_{(11)m} & \cdots & x_{(1\,m)m} \\ \vdots & \ddots & \vdots \\ x_{(m1)m} & \cdots & x_{(mm)m} \end{bmatrix}$$

Figure 5. DRSS design in case of odd sample size

So, we have four judgment ranked sets of size m each:

$$X_{1,j} = min\left\{\left\{x^{(j)}_{(11)}, x^{(j)}_{(22)}, \ldots .x^{(j)}_{(mm)}\right\}, j = 1,2, \ldots, r\right\},$$

$$X_{m,k} = max\left\{\left\{x^{(k)}_{(11)}, x^{(k)}_{(22)}, \ldots .x^{(k)}_{(mm)}\right\}, k = r + 2, \ldots, m\right\},$$

and

$$X_{(r+1),(r+1)} = median\left\{\left\{X_{(1,r+1)}, X_{(2,r+1)}, \ldots .X_{(m,r+1)}\right\}\right\}.$$

The likelihood function corresponding to DRSS scheme that proposed by [9] is given as follows:

<u>Case I: m even ($m = 2r$)</u>

$$L(\theta) = \left[\prod_{j=1}^{r} m f_{1:m}(x_{1,j})\left[1 - F_{1:m}(x_{1,j})\right]^{m-1}\right] \cdot \left[\prod_{k=r+1}^{m} m\, f_{m:m}(x_{m,k})\, \left[F_{m:m}(x_{m,k})\right]^{m-1}\right] \quad (10)$$

where

$f_{1:m}(x_j) = m\, f(x_{1,j})\left[1 - F(x_{1,j})\right]^{m-1}$,

$F_{1:m}(x_{1,j}) = 1 - \left[1 - F(x_{1,j})\right]^{m}$,

$f_{m:m}(x_k) = mf(x_{m,k})\,[F(x_{m,k})]^{m-1}$ and $F_{m:m}(x_{m,k}) = [F(x_{m,k})]^{m}$

Case II: m odd ($m = 2r + 1$)

$$L(\theta) = \left[\prod_{j=1}^{r} m f_{1:m}(x_{1,j})\left[1 - F_{1:m}(x_{1,j})\right]^{m-1}\right] \cdot \left[\prod_{k=r+2}^{m} m f_{m:m}(x_{m,k})\, \left[F_{m:m}(x_{m,k})\right]^{m-1}\right] \cdot \left[\frac{(2r+1)!}{(r)!\,(r)!} f_{r+1:m}(x_{(r+1),(r+1)}) \left(F_{r+1:m}(x_{(r+1),(r+1)})\left(1 - F_{r+1:m}(x_{(r+1),(r+1)})\right)\right)^{r}\right] \quad (11)$$

where

$$F_{r+1:m}(x_{(r+1),(r+1)}) = \sum_{t=r+1}^{m} \binom{m}{t} \left(F(x_{(r+1),(r+1)})\right)^{t} \cdot \left(1 - F(x_{(r+1),(r+1)})\right)^{m-t}$$

3. Estimation of Three-Parameters Weibull Distribution Parameters

This MLE method has a convergence issue and it can also have an unfeasible value so that the location estimate of the three-parameter Weibull model can be greater than the minimum value of the observations . also pointed out that the likelihood function has the unbounded likelihood problem and the location parameter tends to approach the smallest observation. also showed that no stationary point can yield a consistent estimator, which results in no local maximum. Thus, whether a global or a local maximum is sought, the MLE is bound to fail. So we are going to differentiate λ *and* β only.

In this Section MLEs for the unknown parameters of EPGW distribution based on RSS will be reviewed, moreover we will derive MLHs for EPGW distribution based on ERSS and DRSS.

3.1. Estimation Based on RSS

Let $\{X_i^j, i = 1,2,..,n; j = 1,2,..,r\}$ be a ranked set sample with CDF and PDF given in Eq. (1) and Eq. (2), where n is the set size, r is the number of cycles and $m = n\, r$. According to the Eq. (4) the Likelihood function for set sizes m and with r cycles based on RSS is given by

$$L_r(x;\lambda,\beta,\alpha) = \prod_{j=1}^{r}\prod_{i=1}^{n} C_i\, \lambda\beta\left(x_{(i)j} - \alpha\right)^{\beta-1} e^{-\lambda(x_{(i)j}-\alpha)^{\beta}} \cdot \left(1 - e^{-\lambda(x_{(i)j}-\alpha)^{\beta}}\right)^{i-1} \left(e^{-\lambda(x_{(i)j}-\alpha)^{\beta}}\right)^{n-i}$$

The log likelihood function can be derived directly as follows

$$\ell_r(\theta) \propto n\, r \log\lambda + n\, r\log\beta + (\beta - 1)\sum_{j=1}^{r}\sum_{i=1}^{n}\log(x_{(i)j} - \alpha) + \sum_{j=1}^{r}\sum_{i=1}^{n}\left[-\lambda(x_{(i)j} - \alpha)^{\beta}\right] + \sum_{j=1}^{r}\sum_{i=1}^{n}(i-1)\log\left[1 - e^{-\lambda(x_{(i)j}-\alpha)^{\beta}}\right] + \sum_{j=1}^{r}\sum_{i=1}^{n}(n-i)\left[-\lambda(x_{(i)j} - \alpha)^{\beta}\right]$$

The likelihood equations becomes

$$\frac{\partial\ell_{E(e)}}{\partial\lambda} = \frac{n\,r}{\lambda} - \sum_{j=1}^{r}\sum_{i=1}^{n}\left[(x_{(i)j} - \alpha)^{\beta}\right] + \sum_{j=1}^{r}\sum_{i=1}^{n}(i-1)\frac{e^{-\lambda(x_{(i)j}-\alpha)^{\beta}}(x_{(i)j} - \alpha)^{\beta}}{1 - e^{-\lambda(x_{(i)j}-\alpha)^{\beta}}} - \sum_{j=1}^{r}\sum_{i=1}^{n}(n-i)\left[(x_{(i)j} - \alpha)^{\beta}\right]$$

and

$$\frac{\partial\ell_{E(e)}}{\partial\beta} = \frac{n\,r}{\beta} + \sum_{j=1}^{r}\sum_{i=1}^{n}\log x_{(i)j} + \sum_{j=1}^{r}\sum_{i=1}^{n}\left[-\lambda(x_{(i)j} - \alpha)^{\beta}\right]\log(x_{(i)j} - \alpha) + \sum_{j=1}^{r}\sum_{i=1}^{n}(i-1)\frac{e^{-\lambda(x_{(i)j}-\alpha)^{\beta}}\lambda(x_{(i)j} - \alpha)^{\beta}\log(x_{(i)j} - \alpha)}{1 - e^{-\lambda(x_{(i)j}-\alpha)^{\beta}}} + \sum_{j=1}^{r}\sum_{i=1}^{n}(n-i)\left[-\lambda(x_{(i)j} - \alpha)^{\beta}\right]\log(x_{(i)j} - \alpha)$$

3.2. Estimation Based on ERSS

The maximum likelihood function for even set sizes ($m = 2p$) and with r cycles by substitution in Eq. (9) based on ERSS is given by

$$L_{E(e)}(\theta) \propto \prod_{j=1}^{r}\prod_{i=1}^{p} m\,\lambda\beta\,(x_{(1)i,j} - \alpha)^{\beta-1}\left(\exp[-\lambda(x_{(1)i,j} - \alpha)^{\beta}]\right) \cdot \left[1 - e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right]^{m-1} \cdot \left[\prod_{j=1}^{r}\prod_{i=p+1}^{m} m\,\lambda\beta\,(x_{(m)i,j} - \alpha)^{\beta-1}\left(exp[-\lambda(x_{(m)i,j} - \alpha)^{\beta}]\right)^{m}\right]$$

$$L_{E(e)}(\theta) = hm^{rm}\lambda^{rm}\alpha^{rm}\beta^{rm}\left(\prod_{j=1}^{r}\prod_{i=1}^{p}\left(x_{(1)i,j}-\alpha\right)^{\beta-1}\right) \cdot\left(\prod_{j=1}^{r}\prod_{i=1}^{p}\exp\left[-\lambda(x_{(1)i,j}-\alpha)^{\beta}\right]\right) \cdot\left(\prod_{j=1}^{r}\prod_{i=1}^{p}\left[1-e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right]^{m-1}\right) \cdot\left(\prod_{i=1}^{r}\prod_{j=p+1}^{m}(x_{(m)i,j}-\alpha)^{\beta-1}\right) \cdot\left(\prod_{j=1}^{r}\prod_{i=p+1}^{m}\left(exp\left[-\lambda(x_{(m)i,j}-\alpha)^{\beta}\right]\right)^{m}\right)$$

$$\ell_{E(e)}(\theta) = \log\ h + rm\log m + rm\log\lambda + rm\log\beta + (\beta-1)\sum_{j=1}^{r}\sum_{i=1}^{p}\log\left(x_{(1)i,j}-\alpha\right) - \lambda\sum_{j=1}^{r}\sum_{i=1}^{p}\left[(x_{(1)i,j}-\alpha)^{\beta}\right] + \sum_{j=1}^{r}\sum_{i=1}^{p}(m-1)\log\left[1-e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right] + (\beta-1)\sum_{j=1}^{r}\sum_{i=p+1}^{m}\log(x_{(m)i,j}-\alpha) - \lambda\sum_{j=1}^{r}\sum_{i=p+1}^{m}m\left((x_{(m)i,j}-\alpha)^{\beta}\right)$$

and the likelihood equations are given by

$$\frac{\partial\ell_{E(e)}}{\partial\lambda} = \frac{rm}{\lambda} - \sum_{j=1}^{r}\sum_{i=1}^{p}\left[(x_{(1)i,j}-\alpha)^{\beta}\right] + \sum_{j=1}^{r}\sum_{i=1}^{p}(m-1)\frac{e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}(x_{(1)i,j}-\alpha)^{\beta}}{\left(1-e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right)} - \sum_{j=1}^{r}\sum_{i=p+1}^{m}m\left((x_{(m)i,j}-\alpha)^{\beta}\right)$$

and

$$\frac{\partial\ell_{E(e)}}{\partial\beta} = \frac{rm}{\beta} + \sum_{j=1}^{r}\sum_{i=1}^{p}\log\left(x_{(1)i,j}-\alpha\right) - \lambda\sum_{j=1}^{r}\sum_{i=1}^{p}\left[(x_{(1)i,j}-\alpha)^{\beta}\right]log\left(x_{(1)i,j}-\alpha\right) + \sum_{j=1}^{r}\sum_{i=1}^{p}(m-1)\frac{e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\lambda(x_{(1)i,j}-\alpha)^{\beta}\ log(x_{(1)i,j}-\alpha)}{\left(1-e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right)} + \sum_{j=1}^{r}\sum_{i=p+1}^{m}\log(x_{(m)i,j}-\alpha) - \lambda\sum_{j=1}^{r}\sum_{i=p+1}^{m}m\left((x_{(m)i,j}-\alpha)^{\beta}\ log(x_{(m)i,j}-\alpha)\right)$$

When m is odd ($m = 2q+1$) and based on ERSS by substitution in Eq. (8), the maximum likelihood function L is given by

$$L_{E(o)}(\theta) \propto \prod_{j=1}^{r}\prod_{i=1}^{q}m\lambda\beta\ \left(x_{(1)i,j}-\alpha\right)^{\beta-1}\cdot\left(\exp\left[-\lambda(x_{(1)i,j}-\alpha)^{\beta}\right]\right) \cdot\left[1-e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right]^{m-1} \cdot\left[\prod_{j=1}^{r}\prod_{i=q+1}^{m-1}m\ \lambda\beta\ (x_{(m)i,j}-\alpha)^{\beta-1}\left(exp\left[-\lambda(x_{(m)i,j}-\alpha)^{\beta}\right]\right)^{m}\right] \cdot\left[\left(\lambda\beta(x_{((m+1)/2)j}-\alpha)^{\beta-1}\left(\exp\left[-\lambda(x_{((m+1)/2)j}-\alpha)^{\beta}\right]\right)\right) \cdot\left[1-e^{-\lambda(x_{((m+1)/2)j}-\alpha)^{\beta}}\right]^{\frac{m-1}{2}}\left(\exp\left[-\lambda(x_{((m+1)/2)j}-\alpha)^{\beta}\right]\right)^{\frac{m-1}{2}}\right]$$

$$\ell_{E(e)}(\theta) = \log\ k + rm\log m + rm\log\lambda + rm\log\beta + (\beta-1)\sum_{j=1}^{r}\sum_{i=1}^{q}\log\left(x_{(1)i,j}-\alpha\right) - \lambda\sum_{j=1}^{r}\sum_{i=1}^{q}\left[(x_{(1)i,j}-\alpha)^{\beta}\right] + \sum_{j=1}^{r}\sum_{i=1}^{q}(m-1)\log\left[1-e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right] + (\beta-1)\sum_{j=1}^{r}\sum_{i=q+1}^{m-1}\log\left(x_{(m)i,j}-\alpha\right) - \lambda\sum_{j=1}^{r}\sum_{i=q+1}^{m-1}m\left((x_{(m)i,j}-\alpha)^{\beta}\right) + (\beta-1)\log\left(x_{\left(\frac{m+1}{2}\right)j}-\alpha\right) + \frac{m-1}{2}\log\left(1-e^{-\lambda(x_{((m+1)/2)j}-\alpha)^{\beta}}\right) + (\frac{m+1}{2})\left[-\lambda(x_{((m+1)/2)j}-\alpha)^{\beta}\right]$$

and the likelihood equations are given by

$$\frac{\partial\ell_{E(e)}}{\partial\lambda} = \frac{rm}{\lambda} - \sum_{j=1}^{r}\sum_{i=1}^{q}\left[(x_{(1)i,j}-\alpha)^{\beta}\right] + \sum_{j=1}^{r}\sum_{i=1}^{q}(m-1)\frac{e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}(x_{(1)i,j}-\alpha)^{\beta}}{\left(1-e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right)} - \sum_{j=1}^{r}\sum_{i=q+1}^{m-1}m\left((x_{(m)i,j}-\alpha)^{\beta}\right) + \left(\frac{m-1}{2}\right)\frac{e^{-\lambda(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}}(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}}{1-e^{-\lambda(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}}} - (\frac{m+1}{2})\left[(x_{((m+1)/2)j}-\alpha)^{\beta}\right]$$

and

$$\frac{\partial\ell_{E(e)}}{\partial\beta} = \frac{rm}{\beta} + \sum_{j=1}^{r}\sum_{i=1}^{q}\log\left(x_{(1)i,j}-\alpha\right) - \lambda\sum_{j=1}^{r}\sum_{i=1}^{q}\left[(x_{(1)i,j}-\alpha)^{\beta}\right]log\left(x_{(1)i,j}-\alpha\right) + \sum_{j=1}^{r}\sum_{i=1}^{q}(m-1)\frac{e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\lambda(x_{(1)i,j}-\alpha)^{\beta}\ log(x_{(1)i,j}-\alpha)}{\left(1-e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right)} + \sum_{j=1}^{r}\sum_{i=q+1}^{m-1}\log\left(x_{(m)i,j}-\alpha\right) - \lambda\sum_{j=1}^{r}\sum_{i=q+1}^{m-1}m\left((x_{(m)i,j}-\alpha)^{\beta}\ log(x_{(m)i,j}-\alpha)\right) + \log\left(x_{\left(\frac{m+1}{2}\right)j}-\alpha\right) + \left(\frac{m-1}{2}\right)\frac{e^{-\lambda(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}}(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}}{1-e^{-\lambda(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}}} - \left(\frac{m+1}{2}\right)\left[(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}\right] + \left(\frac{m-1}{2}\right)\frac{e^{-\lambda(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}}\lambda(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}\ log\ (x_{(m)i,j}-\alpha)}{1-e^{-\lambda(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}}} + \left(\frac{m+1}{2}\right)\left[-\lambda(x_{\left(\frac{m+1}{2}\right)j}-\alpha)^{\beta}\right]log(x_{(m)i,j}-\alpha)$$

3.3. *Estimation Based on DRSS*

By substitution in Eq. (10) and Eq. (11) based on EPGW distribution the Likelihood function for set sizes m and with r cycles based on DRSS is given by

Case I: m even ($m = 2r$)

$$L_{D(e)}(\theta) = \left[\prod_{j=1}^{r}\left(m^2\lambda\beta\,(x_{(1)i,j}-\alpha)^{\beta-1}\left(e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right)^{m^2}\right)\right]$$
$$\cdot\prod_{k=r+1}^{m}\left(m^2\,\lambda\beta\,(x_{(m)i,j}-\alpha)^{\beta-1}exp\left[-\lambda(x_{(m)i,j}-\alpha)^{\beta}\right]\right)$$
$$\cdot\left(1-e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}\right)^{m^2-1}$$

Then, the associated log-likelihood function is obtained as

$$\ell_{D(e)} = 2m\log m + m\log\lambda + m\log\beta + (\beta-1)\sum_{j=1}^{r}\log(x_{(1)i,j}-\alpha) + (\beta-1)\sum_{k=r+1}^{m}\log(x_{(m)i,j}-\alpha) + m^2\sum_{j=1}^{r}\left(-\lambda(x_{(1)i,j}-\alpha)^{\beta}\right) + \sum_{k=r+1}^{m}\left(-\lambda(x_{(m)i,j}-\alpha)^{\beta}\right) + (m^2-1)\sum_{k=r+1}^{m}\log\left(1-e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}\right).$$

and the likelihood equations are given by

$$\frac{\partial\ell_{D(e)}}{\partial\lambda} = \frac{m}{\lambda} - m^2\sum_{j=1}^{r}\left((x_{(1)i,j}-\alpha)^{\beta}\right) - \sum_{k=r+1}^{m}\left((x_{(m)i,j}-\alpha)^{\beta}\right) + (m^2-1)\sum_{k=r+1}^{m}\frac{e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}(x_{(m)i,j}-\alpha)^{\beta}}{\left(1-e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}\right)}$$

and

$$\frac{\partial\ell_{D(e)}}{\partial\beta} = \frac{m}{\beta} + \sum_{j=1}^{r}\log(x_{(1)i,j}-\alpha) + \sum_{k=r+1}^{m}\log(x_{(m)i,j}-\alpha) + m^2\sum_{j=1}^{r}\left(-\lambda(x_{(1)i,j}-\alpha)^{\beta}\right)\,log\,(x_{(1)i,j}-\alpha) + \sum_{k=r+1}^{m}\left(-\lambda(x_{(m)i,j}-\alpha)^{\beta}\right)log(x_{(m)i,j}-\alpha) + (m^2-1)\sum_{k=r+1}^{m}\frac{e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}\lambda(x_{(m)i,j}-\alpha)^{\beta}\,log\,(x_{(m)i,j}-\alpha)}{\left(1-e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}\right)}$$

Case II: m odd ($m = 2r+1$)

$$L_{D(o)}(\theta) = \left[\prod_{j=1}^{r}\left(m^2\lambda\beta\,(x_{(1)i,j}-\alpha)^{\beta-1}\left(e^{-\lambda(x_{(1)i,j}-\alpha)^{\beta}}\right)^{m^2}\right)\right]$$
$$\cdot\prod_{k=r+2}^{m}\left(m^2\,\lambda\beta(x_{(m)i,j}-\alpha)^{\beta-1}\exp\left[-\lambda(x_{(m)i,j}-\alpha)^{\beta}\right]\right)$$
$$\cdot\left(1-e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}\right)^{m^2-1}$$
$$\cdot\left[\frac{(2r+1)!}{(r)!\,(r)!}\left(\frac{m!}{r!\,r!}\left(\lambda\beta(x_{(r+1),(r+1)}-\alpha)^{\beta-1}\left(\exp\left[-\lambda(x_{(r+1),(r+1)}-\alpha)^{\beta}\right]\right)^{(r+1)}\right)\right.\right.$$
$$\cdot\left(\left(1-e^{-\lambda(x_{(r+1),(r+1)}-\alpha)^{\beta}}\right)^{r}\right)$$
$$\cdot\left(F_{r+1:m}(x_{(r+1),(r+1)})\right)^{r}\left(1-F_{r+1:m}(x_{(r+1),(r+1)})\right)^{r}$$

Then, the associated log-likelihood function is obtained as

$$\ell_{D(e)} = (2m-1)\,log\,m + m\log\lambda + m\log\beta + (\beta-1)\sum_{j=1}^{r}\log(x_{(1)i,j}-\alpha) + (\beta-1)\sum_{k=r+2}^{m}\log(x_{(m)i,j}-\alpha) + m^2\sum_{j=1}^{r}\left(-\lambda(x_{(1)i,j}-\alpha)^{\beta}\right) + \sum_{k=r+2}^{m}\left(-\lambda(x_{(m)i,j}-\alpha)^{\beta}\right) + (m^2-1)\sum_{k=r+2}^{m}\log\left(1-e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}\right) + (\beta-1)\log(x_{(r+1),(r+1)}-\alpha) + (r+1)\left[-\lambda(x_{(r+1),(r+1)}-\alpha)^{\beta}\right] + r\,log\left(1-e^{-\lambda(x_{(r+1),(r+1)}-\alpha)^{\beta}}\right) + r\,log\,F_{r+1:m}(x_{(r+1),(r+1)}) + r\,log\left(1-F_{r+1:m}(x_{(r+1),(r+1)})\right)$$

and the likelihood equations are given by

$$\frac{\partial\ell_{D(e)}}{\partial\lambda} = \frac{m}{\lambda} - m^2\sum_{j=1}^{r}\left((x_{(1)i,j}-\alpha)^{\beta}\right) - \sum_{k=r+1}^{m}\left((x_{(m)i,j}-\alpha)^{\beta}\right) + (m^2-1)\sum_{k=r+2}^{m}\frac{e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}(x_{(m)i,j}-\alpha)^{\beta}}{\left(1-e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}\right)} - (r+1)\left[(x_{(r+1),(r+1)}-\alpha)^{\beta}\right] + r\left(\frac{e^{-\lambda(x_{(r+1),(r+1)}-\alpha)^{\beta}}(x_{(r+1),(r+1)}-\alpha)^{\beta}}{1-e^{-\lambda(x_{(r+1),(r+1)}-\alpha)^{\beta}}}\right) + F_{\lambda}\left(\frac{1-2F_{r+1:m}(x_{(r+1),(r+1)})}{F_{r+1:m}(x_{(r+1),(r+1)})\left(1-F_{r+1:m}(x_{(r+1),(r+1)})\right)}\right)$$

and

$$\frac{\partial \ell_{D(e)}}{\partial \beta} = \frac{m}{\beta} + \sum_{j=1}^{r} \log(x_{(1)i,j} - \alpha) + \sum_{k=r+1}^{m} \log(x_{(m)i,j} - \alpha) + m^2 \sum_{j=1}^{r} \left(-\lambda(x_{(1)i,j} - \alpha)^{\beta}\right) log\ (x_{(1)i,j} - \alpha) + \sum_{k=r+2}^{m} \left(-\lambda(x_{(m)i,j} - \alpha)^{\beta}\right) log(x_{(m)i,j} - \alpha) + (m^2 - 1) \sum_{k=r+2}^{m} \frac{e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}} \lambda(x_{(m)i,j} - \alpha)^{\beta} \log(x_{(m)i,j} - \alpha)}{\left(1 - e^{-\lambda(x_{(m)i,j}-\alpha)^{\beta}}\right)} + \log(x_{(r+1),(r+1)} - \alpha) + (r + 1)[-\lambda(x_{(r+1),(r+1)} - \alpha)^{\beta}]\ log\ (x_{(r+1),(r+1)} - \alpha) + r \left(\frac{e^{-\lambda(x_{(r+1),(r+1)}-\alpha)^{\beta}} \lambda(x_{(r+1),(r+1)} - \alpha)^{\beta}\ log\ (x_{(r+1),(r+1)} - \alpha)}{1 - e^{-\lambda(x_{(r+1),(r+1)}-\alpha)^{\beta}}}\right) + F_{\beta} \left(\frac{1 - 2F_{r+1:m}(x_{(r+1),(r+1)})}{F_{r+1:m}(x_{(r+1),(r+1)})\ \left(1 - F_{r+1:m}(x_{(r+1),(r+1)})\right)}\right)$$

4. Simulation Study

In this section, a simulation study will be conducted to compare the maximum likelihood estimates of the Three-Parameters Weibull distribution ratio and shape parameters based on different sampling schemes. The simulation is suitable for 10,000 repetitions and different sample sizes, m = {10,15,20,25}. For different parameter values EPGW (λ, β) = { (0.5, 0.5), (0.5, 1.5), (1.5, 1.5)} for simulation of the statistical software R [20]. Use MSE and efficiency standards to compare the λ and β estimators proposed using SRS, RSS, ERSS and DRSS. Calculate the efficiency of all estimators based on SRS relative to MLE. The efficiency of the estimator is defined as

$$eff(\hat{\theta}_1, \hat{\theta}_2) = \frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)},$$

if $eff(\hat{\theta}_1, \hat{\theta}_2) > 1$, then $\hat{\theta}_2$is better than $\hat{\theta}_1$.

Table 1and Table 2 lists the results of bias and MSE for different estimators, table (3) lists the efficiency results, and Figure (6-11) shows the simulation results. The following conclusions can be observed from Table (1-2)

1) It is small biases in almost all cases
2) The MSE of the (λ, β) estimator based on SRS data is greater than the MSE of the estimator based on RSS, ERSS and DRSS data in all cases. (See Figure 6).

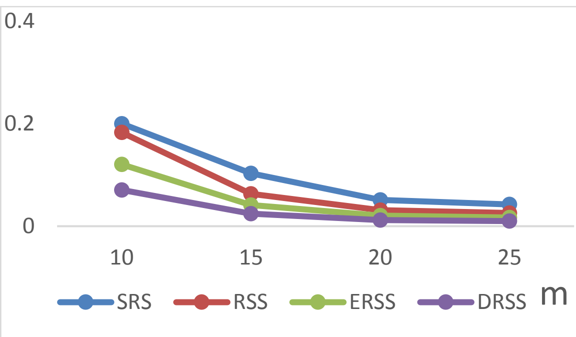

Figure 6. MSEs of the estimators for $\lambda = 0.5$

3) As the set size increases, the MSE of all estimators based on SRS, RSS, ERSS and DRSS will decrease in almost all cases. (See Figure 7).

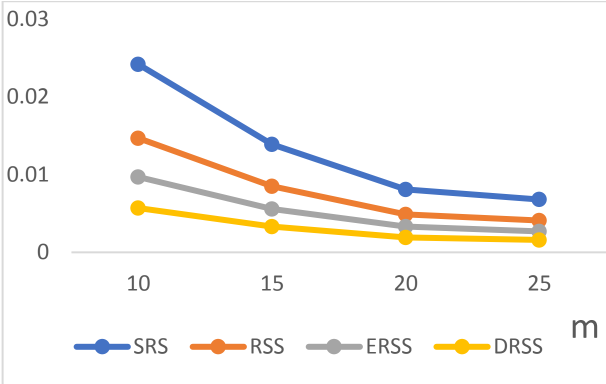

Figure 7. MSEs of the estimators based on SRS, RSS, ERSS, and DRSS at ($\beta = 0.5$)

4) the MSE of all estimators based on SRS, RSS, ERSS and DRSS decreases as the value of λ increases (see Figure 8). As the value of β increases, the MSE of all estimates based on SRS, RSS, ERSS and DRSS will decrease (see Figure 9).MSEs of the estimators for (λ) based on DRSS have the smallest
5) In almost all cases, The MSEs of the MRSS scheme based on estimators (λ and β) are smaller than the MSEs of the RSS estimators.

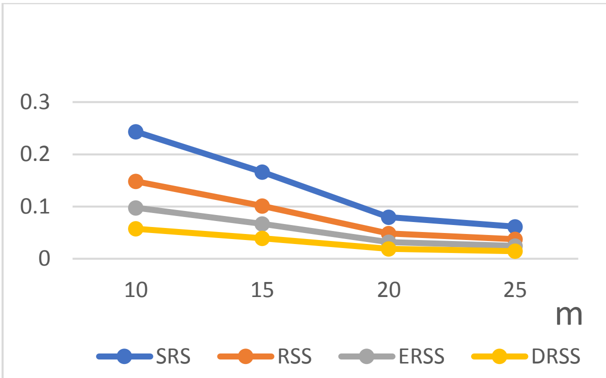

Figure 8. MSEs of the estimators based on SRS, RSS, ERSS, and DRSS at $\lambda = 1.5$

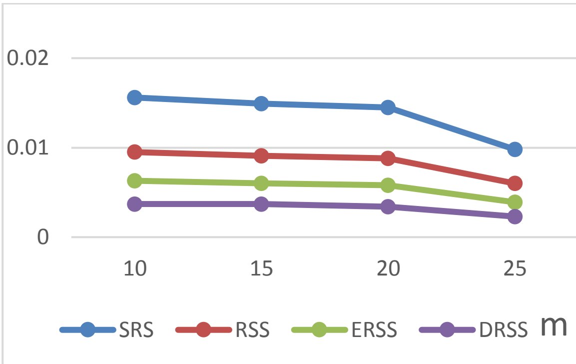

Figure 9. MSEs of the estimators based on SRS, RSS, ERSS, and DRSS at $\beta = 1.5$

From Table 3, it can be observed that:

6) As the set size increases, the efficiency of all estimators based on RSS, ERSS and DRSS will increase in almost all cases. (See Figure 10).

7) in all cases, Efficiencies of the estimators for λ based on DRSS has the highest efficiency, (See Figure 10).

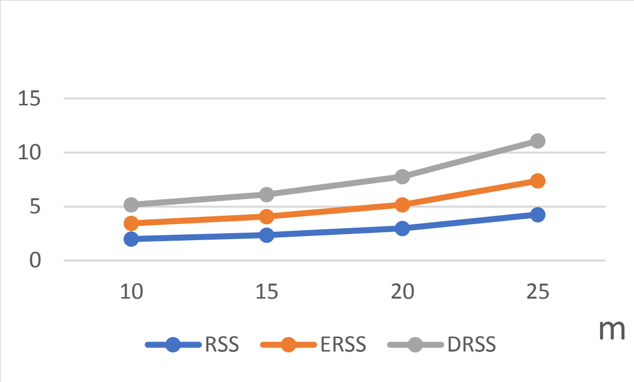

Figure 10. Efficiencies of the estimators based on RSS, ERSS, and DRSS at $\lambda = 0.5$

8) in all cases, Efficiencies of the estimators for β based on DRSS have the largest efficiencies (See Figure 11).

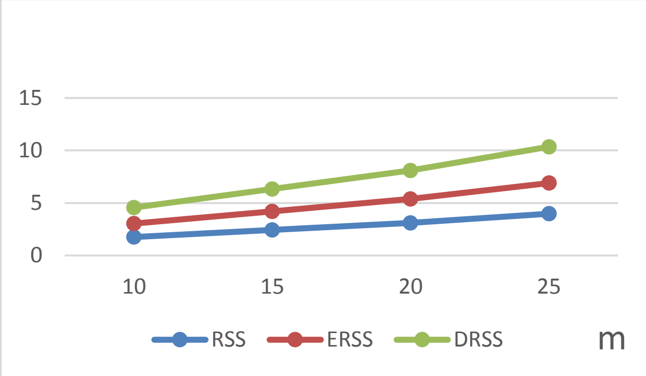

Figure 11. Efficiencies of the estimators based on RSS, ERSS, and DRSS at $\beta = 0.5$

9) In almost all cases, The efficiency of (λ and ∝) estimators on ERSS is greater than that of RSS estimators.

Table 1. Biases of the estimators for (λ, β) based on SRS, RSS, ERSS, and DRSS

Three-Parameters Weibull (λ, β)	m	SRS		RSS		MRSS		NRSS	
		λ	β	λ	β	λ	β	λ	β
(0.5,0.5)	10	-0.3290	-0.9610	-0.0923	0.4495	0.3330	0.0840	-0.2387	-0.1775
	15	-0.1080	-0.7755	-0.0791	0.4014	0.2357	-0.0696	-0.2189	-0.1639
	20	-0.0887	-0.6325	-0.0477	0.2472	0.1591	-0.0430	-0.1837	-0.1031
	25	-0.0440	-0.1280	-0.4442	0.1939	0.1251	-0.0273	-0.1530	-0.0808
(0.5,1.5)	10	-0.3948	-1.1532	-0.1107	0.5394	0.3996	0.1008	-0.2864	-0.213
	15	-0.1296	-0.9306	-0.0949	0.48168	0.28284	-0.0832	-0.2626	-0.1968
	20	-0.1064	-0.759	-0.0572	0.29664	0.1902	-0.0516	-0.2204	-0.1372
	25	-0.0528	-0.1536	-0.5334	0.23268	0.1512	-0.0326	-0.183	-0.0996
(1.5,1.5)	10	-0.2278	-0.6653	-0.0639	0.3112	0.2305	0.0582	-0.1653	-0.1229
	15	-0.0748	-0.5369	-0.0548	0.2779	0.1632	-0.0482	-0.1515	-0.1135
	20	-0.0614	-0.4379	-0.0330	0.1711	0.1101	-0.0298	-0.1272	-0.0714
	25	-0.0305	-0.0886	-0.3075	0.1342	0.0866	-0.0189	-0.1059	-0.0559

Table 2. MSEs of the estimators for (λ, β) based on SRS, RSS, ERSS, and DRSS

Three-Parameters Weibull	m	SRS		RSS		MRSS		NRSS	
		λ	β	λ	β	λ	β	λ	β
EPGW (0.5,0.5)	10	0.1999	0.0242	(0.5,0.5)	0.0147	0.1204	0.0097	0.0707	0.0057
	15	0.1033	0.0139	0.0629	0.0085	0.0415	0.0056	0.0243	0.0033
	20	0.0512	0.0081	0.0312	0.0049	0.0205	0.0033	0.0121	0.0019
	25	0.0424	0.0068	0.0258	0.0041	0.0170	0.0027	0.0100	0.0016
EPGW (0.5,1.5)	10	0.1683	0.0156	(0.5,1.5)	0.0095	0.0675	0.0063	0.0397	0.0037
	15	0.0924	0.0149	0.0563	0.0091	0.0371	0.0234	0.0218	0.0037
	20	0.0283	0.0145	0.0172	0.0088	0.0114	0.0058	0.016	0.0034
	25	0.0215	0.0098	0.0131	0.0060	0.0086	0.0039	0.0091	0.0023
EPGW (1.5,1.5)	10	0.2434	0.0395	(1.5,1.5)	0.0240	0.0977	0.0159	0.0574	0.0093
	15	0.1662	0.0293	0.1012	0.0178	0.0667	0.0118	0.0392	0.0069
	20	0.0798	0.0259	0.0486	0.0158	0.0320	0.0104	0.0188	0.0061
	25	0.06150	0.01870	0.03744	0.01139	0.02468	0.00751	0.0149	0.0044

Table 3. Efficiencies of the estimators for (λ, β) based on RSS, ERSS, and DRSS

Three-Parameters Weibull (λ, β)	m	RSS		MRSS		NRSS	
		λ	β	λ	β	λ	β
EPGW (0.5,0.5)	10	1.6425	(0.5,0.5)	2.8473	2.8473	4.2731	4.2758
	15	2.3625	2.1013	4.0955	3.6427	6.1463	5.4703
	20	3.2555	2.9855	5.6435	5.1754	8.4695	7.7721
	25	3.8755	3.1435	6.7183	5.4493	10.0825	8.1834
EPGW (0.5,1.5)	10	1.9855	(0.5,1.5)	3.4419	3.0371	5.1655	4.5609
	15	2.3533	2.4253	4.0795	4.2043	6.1224	6.3137
	20	2.9852	3.1112	5.1749	5.3933	7.7663	8.0993
	25	4.2556	3.9755	7.3772	6.8916	11.0714	10.3493
EPGW (1.5,1.5)	10	2.2522	(1.5,1.5)	3.9042	4.4347	5.8593	6.6597
	15	2.8550	2.9862	4.9492	5.1767	7.4276	7.7739
	20	3.2156	3.6578	5.5743	6.3409	8.3657	9.5222
	25	3.9980	4.1690	6.9306	7.2271	10.4012	10.8530

5. Conclusions

According to the numerical results, it can be concluded that compared with RSS and its modification schemes, MSE-based SRS data has the largest MSE. It can be noticed that in almost all cases, the MSE decreases as the setting size increases, and the efficiency increases as the setting size increases. This research shows that ERSS is better than RSS. Moreover, DRSS technology has advantages over other sampling schemes. In almost all cases, DRSS has the smallest MSE and the highest efficiency. Generally, estimators based on DRSS, EERS, and RSS based on RSS technology are more effective than estimators based on SRS technology.

Conflict of Interest Statement

The authors declare no conflict of interest.

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