Computational Research Progress in Applied Science & Engineering

ISSN 2423-4591

CRPASE Vol. 05(02), 58-62, June 2019

Multifocous Image Fusion Using Laplacian Pyramid Technique Based on Alpha Stable Filter

Ias Sri Wahyuni[a*], Rachid Sabre[b]

[a] *Laboratory LE2I, University of Burgundy, Dijon, France*
[b] *Laboratory Biogeosciences University of Burgundy /Agrosup Dijon, France*

Keywords

Laplacian Pyramid,
Gaussian Filter,
Alpha Stable Filter,
Multi focus image fusion.

Abstract

Multifocus image fusion is the process of combining several images from the same scene with different focused object into an image with better focus across all area. The blurred part of multifocus images is a low-pass filtering process and Gaussian filter is often used in this process. Mutlifocus image can be fused using Laplacian Pyramid (LP). LP is computed using two basic operation: reduce and expand that involve low-pas filter. The filter used in LP is Gaussian. In this paper, we propose a multifocus image fusion using LP based on a class of Alpha Stable filters. We apply this method to two cases multifocus imges: (i) multifocus images where the blurred part is generated using Gaussian filter and (ii) multifocus images with blurred part is from Alpha Stable filter and non-Gaussian. And the proposed method give the better results on both cases.

1. Introduction

Imaging cameras usually have only a finite depth of field. It is often not possible to get an image with contains all relevant object 'in focus' so that one scene of image can be taken into set of images with different focus object of every image, while the rest objects of are blurred. We can use image fusion method to obtain all focused objects in an image. Blurring or defocusing is a low-pass filtering process. So that the blurred image can be described as the result of convolving the focus image with the blurring filter. A two-dimensional Gaussian filter is often used to approximate the blurring filter [1, 2].

As we know that Gaussian density distribution is a particular cases of alpha-stable distribution (α=2). Alpha stable distribution best describes noises that are impulsive in nature. Alpha stable distribution has been used to model many phenomena where the Gaussian is not a reasonable choice (when the variance is infinite). Noises of such class contain sharp or occasional burst spikes. Impulsive noises, which can be modeled with alpha stable distributions include atmospheric noise in radio links, switching transients and accidental hits in telephone lines [3]. Alpha Stable distribution have also modeled phenomena in economics [4], physics [5], electrical engineering [6], and image processing [7].

One of image fusion technique is Laplacian Pyramid (LP) image fusion. Laplacian Pyramid which is originally developed by [8] have been used to analyze images at multiple scales for a broad range of applications such as image compression [8], texture synthesis [9], and image fusion [10, 11] 2011]. LP is computed using two basic operations: reduce and expand based on low-pass filter. The filter used in LP is Gaussian. In this paper we propose a method for fusing images based on a class of Alpha Stable filters ($1.3 \leq \alpha \leq 2$) as filter. Discrete Wavelet Transform fusion is used as integration rule in the core of LP fusion method.

This paper is organized as follow: Section 2 describes the Alpha Stable filter that used in the fusion process. In section 3, we provide an explanation about Laplacian Pyramid fusion method. Section 4 presents the proposed method, Laplacian Pyramid fusion method using Alpha Stable filter and experimental result. And section 5 gives conclusion of this work.

2. Alpha Stable Distribution

The Alpha-stable distribution is widely used in the processing of impulsive or spiky signals. It also has been applied in image processing field. [12] Models the sea clutter in SAR images using alpha stable distribution for ship detection while [13] removes speckle noise using alpha stable based Bayesian algorithm in the wavelet domain. Furthermore, alpha stable distribution is also used in image segmentation [14] and compressive image fusion [15]. Both [13], [14], and [15] and Wan employ alpha stable in wavelet

* Corresponding Author:
E-mail address: rachid.sabre@agrosupdijon.fr

Received: 22 March 2019; Accepted: 21 May 2019

domain. This section provides a brief of the alpha-stable distribution.

2.1. Alpha Stable Filter

In this work, we deal with filter generated by alpha stable distribution. The symmetric α-stable (SαS) distribution is best defined by its characteristic function (Eq. (1))

$$\varphi(\omega) = \exp\left(j\delta\omega - \gamma|\omega|^{\alpha}\right) \tag{1}$$

where

$\alpha =$ the characteristic exponent, $0 < \alpha \le 2$
$\delta =$ the location parameter, $-\infty < \delta < \infty$
$\gamma =$ the dispersion of the distribution.

As we work with image, two dimensional case, alpha-stable filter used is from bivariate stable distribution. Bivariate stable distributions much like the univariate stable distributions are characterized by the stability property and the generalized central limit theorem [16]. However, It is more difficult to describe. Bivariate stable distribution appropriates for modeling signals and noise.
The characteristic function of bivariate isotropic α -stable has the form Eq .(2)

$$\varphi(w_1, w_2) = \exp\left(j(\delta_1 w_1 + \delta_2 w_2) - \gamma|w|^{\alpha}\right) \tag{2}$$

where $w = (w_1, w_2)$ and $|w| = \sqrt{w_1^2 + w_2^2}$.

The parameters δ_1, δ_2 are the location parameters. The distribution is isotropic with respect to the point (δ_1, δ_2). Note that the two marginal distributions of the isotropic stable distribution are SαS with parameters $(\delta_1, \gamma, \alpha)$ and $(\delta_2, \gamma, \alpha)$. The bivariate isotropic Cauchy and Gaussian distributions are special cases for $\alpha = 1$ and $\alpha = 2$, respectively.

In this paper we consider that the isotropic stable distribution is centered at origin, $(\delta_1, \delta_2) = (0,0)$. As in the case of the univariate SαS density function, when $\alpha \ne 1$ or $\alpha \ne 2$, no closed form expressions exist for the density function of the bivariate stable random variable. By using polar coordinate $r = |x| = \sqrt{x_1^2 + x_2^2}$, the density function can be written as $f_{\alpha,\gamma}(x_1, x_2) = f_{\alpha,\gamma}(r)$, and can be expressed in a power series expansion form Eq. (3)

$$f_{\alpha,\gamma}(r) = \begin{cases} \dfrac{1}{\pi^2\gamma^{2/\alpha}} \displaystyle\sum_{k=1}^{\infty} \frac{2^{\alpha k}(-1)^{k-1}}{k!} \left(\Gamma(\alpha k/2+1)\right)^2 \sin\left(\frac{k\alpha\pi}{2}\right)\left(\frac{r}{\gamma^{1/\alpha}}\right)^{-\alpha k-2} & \text{for } 0<\alpha<1 \\ \dfrac{\gamma}{2\pi\left(r^2+\gamma^2\right)^{3/2}} & \text{for } \alpha=1 \\ \dfrac{1}{\pi\alpha\gamma^{2/\alpha}} \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k+1}(k!)^2}\Gamma\left(\frac{2k+2}{\alpha}\right)s\left(\frac{r}{\gamma^{1/\alpha}}\right)^{2k} & \text{for } 1<\alpha<2 \\ \dfrac{1}{4\pi\gamma}\exp\left[-\dfrac{r^2}{4\gamma}\right] & \text{for } \alpha=2. \end{cases} \tag{3}$$

2.2. Blurred or Defocused Image

Blurring or defocusing is a low-pass filtering process. So that the blurred image can be described as the result of convolving the focus image with the blurring function. The blurring function can be modeled using physical optics. Very often, a two-dimensional Gaussian function is used to approximate the physical model [17]. The blurred image $g_b(i,j)$ can be obtained as Eq. (4)

$$g_b(i, j) = h(i, j) \otimes g_f(i, j) \tag{4}$$

where $h(i,j)$ is a blurring function and $g_f(i,j)$ is focused image.

3. Laplacian Pyramid Fusion Method

The Laplacian pyramid was first introduced by [3]. The basic idea of Laplacian Pyramid fusion method is to perform a pyramid decomposition on each source image, then integrate all these decompositions to form a composite representation, and finally reconstruct the fused image by performing an inverse pyramid transform.

Laplacian pyramid decomposition is done by taking the difference of levels in the Gaussian pyramid. This pro-cess involves two main operations: reduce and expand. Schematic diagram of the Laplacian Pyramid fusion meth-od is shown in Figure 1.

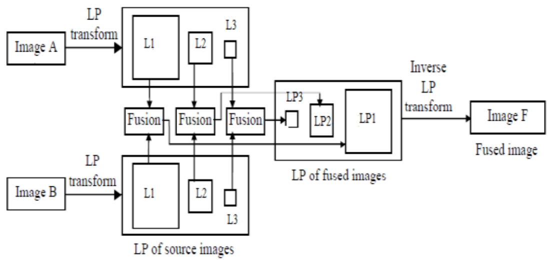


Figure 1. Scheme of LP Fusion Method

Selection fusion rule used in this Laplacian Pyramid is Discrete Wavelet Transform (DWT) fusion with Choose-Max Absolute [18].

3.1. Gaussian Pyramid Decomposition

Suppose g_0 is the original image with size RxC. This image becomes the bottom or zero level of pyramid. Pyramid level 1 contains image g_1, which is reduce and low-pass filtered version of g_0. Pyramid level 2, g_2, is obtained by applying reduce and low-pass filtered version of g_1. The level-to-level process is as Eq. (5)

$$g_l = reduce(g_{l-1})$$

which means, for level $0 < l < N$ and nodes $i, j, 0 < i < C_l, 0 < j < R_l$

$$g_l(i, j) = \sum_{m=-2}^{2}\sum_{n=-2}^{2} w(m,n) g_{l-1}(2i+m, 2j+n) \tag{5}$$

N refers to the number of level in the pyramid and C_l and R_l are the dimension of the lth level. $w(m,n)$ is generating kernel generated from gaussian distribution.

Iterative pyramid generation is equivalent to convolving the image g_0 with a set of equivalent functions h_l:

$$g_l = h_l \otimes g_0$$

or

$$g_l = \sum_{m=-M_l}^{M_l} \sum_{n=-M_l}^{M_l} h_l(m,n) g_0(i2^l + m, j2^l + n) \qquad (6)$$

The sequence image g_0, g_1, g_2, …, g_N is called Gaussian pyramid. The variance of this Gaussian density approximates to 1.

A function expand is the reverse of funtion *reduce*. Its effect is to expand an (M+1)-by-(N+1) array into a (2M+1)-by-(2N+1) array by interpolating new node values between the given values. Thus, expand applied to array g_l of the Gaussian pyramid would yield an array $g_{l,1}$ which is the same size as g_l. Let $g_{1,n}$ be the result of expanding g_l n times. Then

$$g_{l,0} = g_0$$

and

$$g_{l,n} = expand\,(g_l, n - 1)$$

by expand it means, for level $0 < l \leq N$ and $0 \leq n$ and nodes $i, j, 0 < i < C_{l-n}, 0 < j < R_{l-n}$,

$$g_{l,n}(i,j) = 4 \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n)\, g_{l,n-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right) \qquad (7)$$

where

$$g_{l,n-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right) = \begin{cases} g_{l,n-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right), \; for \; \frac{i-m}{2}, \frac{j-n}{2} \text{ integer} \\ 0, otherwise \end{cases}$$

3.2. Laplacian Pyramid Generation

The laplacian pyramid is a sequence of error images L_0, L_1, L_2, L_3, , L_N. Each is the difference between two levels of the Gaussian pyramid

$$L_l = g_l - expand(g_{l+1}, 1) \text{ for } l = N-1, N-1, \ldots 0 \qquad (8)$$

The equation (8) can be written as follow

$$L_l = g_l - g_{l+1,1} \qquad (9)$$

and for L_N,

$$L_N = g_N.$$

The original image, g_0, can be obtained by expanding then summing all the levels of Laplacian pyramid Eq.(10)

$$g_l = \begin{cases} L_l + expand(g_{l+1}, 1) & \text{for } \; l = N-1, N-2, \ldots, 0 \\ L_N & \text{for } \; l = N \end{cases} \qquad (10)$$

4. Proposed Method: Laplacian Pyramid Fusion with Alpha Stable Filter

Gaussian filter is often used in LP fusion method. In this work, the filter used for LP is from class of Alpha Stable as described in section 2. First, to generate a blurred image g we use the convolution of Alpha Stable filter and reference image g_f:

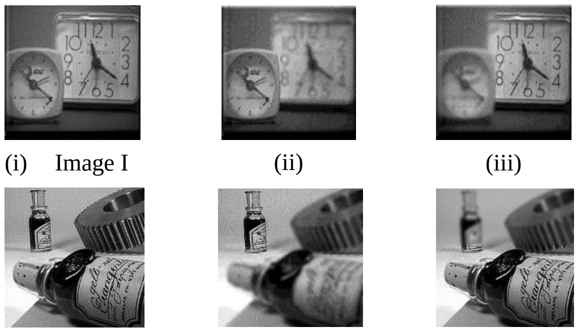
$$g(i,j) = \begin{cases} \sum_{n=-2}^{2} \sum_{m=-2}^{2} h(m',n') g_f(i-m', j-n'), (i,j) \in blurred\ area \\ g_f(i,j), (i,j) \in object\ focus\ area \end{cases} \qquad (11)$$

where $h(m',n')$ is Alpha Stable filter.

The generating kernel, $w\ (m,\ n)$, of the reduce and expand function of this LP (Eq. (5) and Eq. (7)) is generated from Alpha Stable distribution ($1.3 \leq \alpha \leq 2$). We distinguish two cases the blurring mutifocus images with Gaussian filter (α=2) and blurring multifocus images with Alpha Stable filter ($1.3 \leq \alpha < 2$).

4.1. Case 1: Fusion When the Blurred Multifocus Images is Gaussian

We applied the method on the several sets of multi focus images. In this paper, as limited number of pages, we present only two sets of multi focus images. Figure 2 shows the reference images, (i) and (iv) and multi focus images, (ii), (iii), (v), and (vi).



(i) Image I (ii) (iii)

(iv) Image II (v) (vi)

Figure 2. References and multi focus images

In first experiment, multi focus images here are generated by low-pass filtering using Gaussian filter on the reference images. The images were fused using laplacian pyramid image fusion method based on alpha stable filter (α = 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, and 1.9, 2 and $\gamma = 1$) with DWT method.

We analyze the performance of the results using quantitative analysis root mean square error (RMSE) which gives the information how the pixel values of fused image deviate from the reference image. Let $F\,(i,j)$ be the gray level intensity of pixel $(i,\ j)$ of the fused image and $R\,(i,j)$ be the gray level intensity of pixel $(i,\ j)$ of the reference image. RMSE between the reference image and fused image is computed as Eq. (12)

$$RMSE = \sqrt{\frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} [R(i,j) - F(i,j)]^2} \qquad (12)$$

where m x n is the size of the input image and i, j represents to the pixel locations. A smaller value of RMSE shows good fusion result. If the value of RMSE is 0 then it means the fused image is exactly the same as reference image.

Table 1 shows the results of RMSE of LP fusion method based on Alpha Stable filter for Gaussian blurred. From the table, Alpha Stable filter (α = 1.7) gives better fusion result than Gaussian filter (α = 2) usually used in LP fusion.

Table 1. *RMSE of Case 1*

α	Image I	Image II
1,3	10,15983	13,64768
1,4	10,03297	13,42277
1,5	9,94023	13,32989
1,6	9,88894	13,28762
1,7	**9,87448**	**13,27315**
1,8	9,87869	13,28181
1,9	9,89662	13,29844
2	9,930595	13,31834

4.2. Case 2: Fusion When The Blurred Multi Focus Images is Alpha Stable Non-Gaussian

In this section we are interested in the blurred images generated by a non-Gaussian filter. We want to know if the LP fusion by Alpha Stable filter also gives good results. In this experiment, multifocus images used are generated using Alpha Stable filter with α = 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, and 1.9 for blurred area on the reference images. Again, we fuse these multifocus images using LP fusion method based on Alpha Stable filter with α = 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, and 1.9, 2 and $\gamma = 1$ with DWT method.

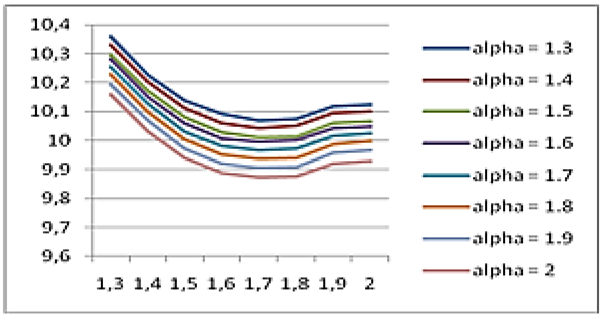


Figure 3. *RMSE of fused image I for different α values of both blurring filter and filter in LP.*

Figure 3 shows the RMSE of fused image using LP for set of multifocus image I (watch) where the blurred part. The horizontal-axis are α values of filter in LP and the colour curves are RMSE of differents α values of blurring filter. From Fig. 3. The best result is obtained when α of Alpha Stable filter in LP is 1.7 for all α values of blurring Alpha Stable filter.

4.3. Theoritical Explanation of The Results

Lemma: The alpha stable distribution is a direct generalization of the Gaussian distribution which shares many of its familiar property: the convolution stability property which means the convolution of two stable distribution is also stable and the central limit theorem which means that every stable random variable may be expressed as a limit, in distribution, of a normalized sum of independents and identically distributed random variables.

Demonstration of this lemma is given in reference [6].

We show that first level using reduce function in LP by Alpha Stable filter on blurred images of Alpha Stable is similar to reduce function in LP with another filter.

$$\begin{aligned} g_1(i,j) &= \sum_{m=-2}^{2}\sum_{n=-2}^{2} w(m,n)g(2i+m,2j+n) \\ &= \sum_{m=-2}^{2}\sum_{n=-2}^{2} w(m,n)\sum_{m'=-2}^{2}\sum_{n'=-2}^{2} h(m',n')g(2i+m-m',2j+n-n') \\ &= \sum_{m=-2}^{2}\sum_{n=-2}^{2}\sum_{m'=-2}^{2}\sum_{n'=-2}^{2} w(m,n)h(m',n')g(2i+m-m',2j+n-n') \end{aligned} \quad (13)$$

subtitute $m''=m-m'$ and $n''=n-n'$ to (13).

we obtain:

$$g_1(i,j) = \sum_{m=-2}^{2}\sum_{n=-2}^{2}\sum_{m''=m-2}^{m+2}\sum_{n''=n-2}^{n+2} w(m,n)h(m-m'',n-n'')g(2i+m'',2j+n'') \quad (14)$$

Eq. (14) can be written

$$\begin{aligned} g_1(i,j) &= \sum_{n''=-4}^{4}\sum_{m''=-4}^{4}\sum_{\substack{q=-2 \\ n''-2\le q\le n''+2}}^{2}\sum_{\substack{p=-2 \\ m''-2\le p\le m''+2}}^{2} w(p,q)h(p-m'',q-n'')g(2i+m'',2j+n'') \\ &= \sum_{n''=-4}^{4}\sum_{m''=-4}^{4} J(m'',n'')g(2i+m'',2j+n'') \end{aligned}$$

(15)

5. Conclusions

Mutifocus image fusion using Laplacian Pyramid fusion method based on Alpha Stable filter (α = 1.7 and $\gamma = 1$) for both cases: (i) multifocus images where the blurred part is generated using Gaussian filter and (ii) multifocus images with blurred part is from Alpha Stable filter and non-Gaussian, gives better result than an ordinary LP fusion with Gaussian filter.

References

[1] A.Petland, A New Sense for Depth of Field, IEEE Transactions on Pattern Analysis and Machine Intelligent 9 (1987) 523–531.

[2] M. Subbarao, Efficient Depth Recovery through Inverse Optics, Machine Vision for Inspection and Measurement, edited by H. Freeman, Academic Press, 1989.

[3] L. Mihaylova, P. Brasnett, A. Achim, D. Bull, N. Canagarajah, Particle Filtering with Alpha-Stable Distributions, IEEE Workshop on Statistical Signal Processing, (2005) 381–386.

[4] G. Hannsgen, Infinite-Variance, Alpha-Stable Shocks in Monetary SVAR, International Review of Applied Economics, Taylor and Francis Group 26 (2012) 755–786.

[5] Z.R. Zhang, Y.M. Kang, Y. Xie, Stochastic Resonance in a Simple Threshold Sensor System with Alpha Stable Noise, Com-munications in Theoretical Physics, The Smithsonian/NASA As-trophysics Data System 61 (2014) 578–582.

[6] N. Azzaoui, L. Clavier, R. Sabre, Path Delay Model Based on α-Stable Distribution for the 160GHz Indoor Channel, Proceed-ing of Global Telecommunications Conference 3 (2003) 1638–1643.

[7] A. Achim, P. Tsakalides, A. Bezerianos, SAR Image Denoising via Bayesian Wavelet Shrinkage Based on Heavy-tailed Modeling, IEEE Transaction on Geoscience and Remote Sensing 41 (2003) 1773–1784.

[8] P.J. Burt, E.H. Adelson, The Laplacian Pyramid as a Compact Image Code, IEEE Transactions on communication 31 (1983) 532–540.

[9] D.J. Heeger and J. R. Bergen, Pyramid Based Texture Analysis/Syntesis, in Computer Graphic Proceedings (1995) 229–238.

[10] P.J. Burt, R.J. Kolezynski, Enhanced Image Capture Through Fusion, International Conference on Computer Vision (1993) 173–182.

[11] W. Wang, F. Chang, A multi-focus image fusion method based on laplacian pyramid, Journal of Computers 6 (2011) 2559–256.

[12] C. Wang, M. Liao, X. Li, Ship Detection in SAR Image Based on the Alpha-stable Distribution, Sensors 8 (2008) 4948–4960.

[13] A. Achim , A. Bezerianos , P. Tsakalides, An Alpha-Stable Based Bayesian Algorithm For Speckle Noise Removal In The Wavelet Domain, in 5th IEEE-EURASIP Biennial International Workshop on Nonlinear Signal and Image Processing, Baltimore, USA, 2001.

[14] T. Wan, N. Canagarajah, A. Achim, A Statistical Mul-tiscale Image Segmentation via Alpha-Stable Modeling, IEEE International Conference on Image Processin. 4 (2007) 357–360.

[15] T. Wan, N. Canagarajah, A. Achim, Compressive Image Fusion, IEEE International Conference on Image Processing (2008) 1308–1311.

[16] C.L. Nikias, M. Shao, Signal Processing with Alpha-Stable Distributions and Applications (Adaptive and Learning Systems for Signal Processing, Communications and Control Series, 1995.

[17] S.K. Nayar, Shape from Focus System, IEEE Computer Vision and Pattern Recognition, Proceedings CVPR, 1992.

[18] Q. Guihong, Z. Dali, Y. Pingfan, Medical image fusion by wavelet transform modulus maxima, [Online]. Opt. Express 9 (2001) 184–190.